Mathematical Developments in Oil – and Gas Exploration and Production

by

Hennie Poulisse

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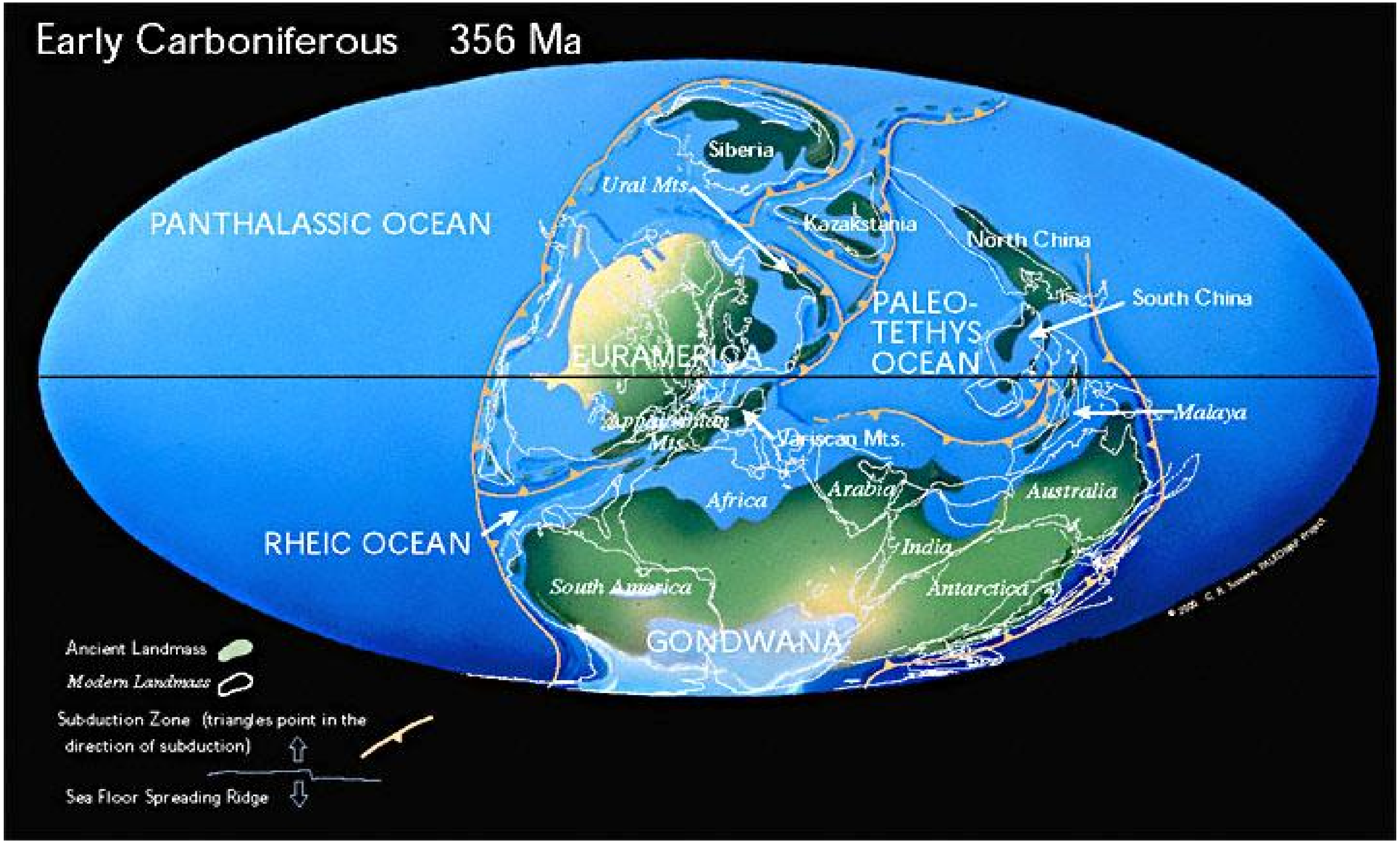
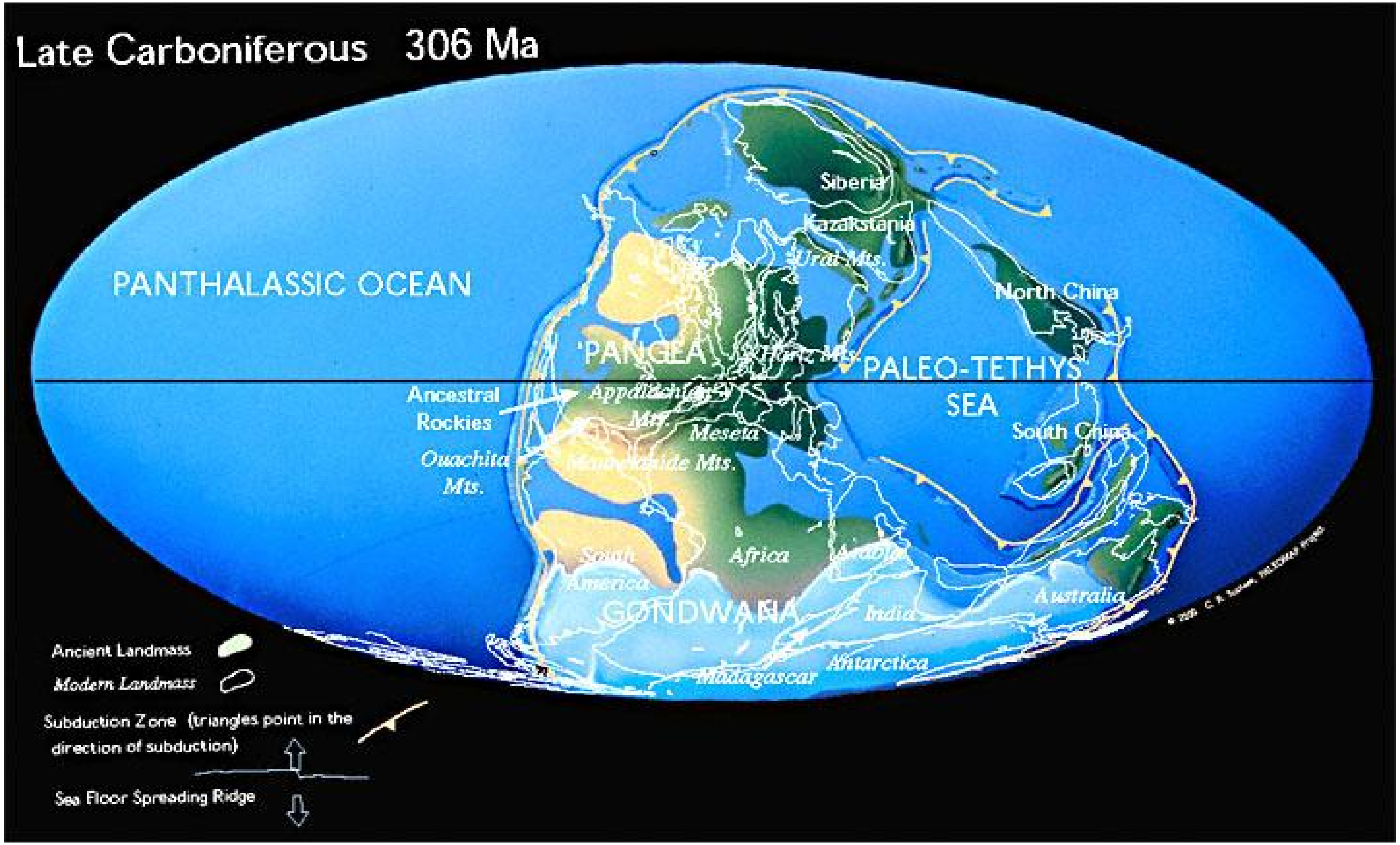
2. Exploration
3. Production
4. Production Operations
5. A First Paradigm Shift
6. A Second Paradigm Shift
7. Algebraic Encounters of the First Kind
8. The CoCOil Research Programme

'Difficult Mission'
Site-seeing Tour

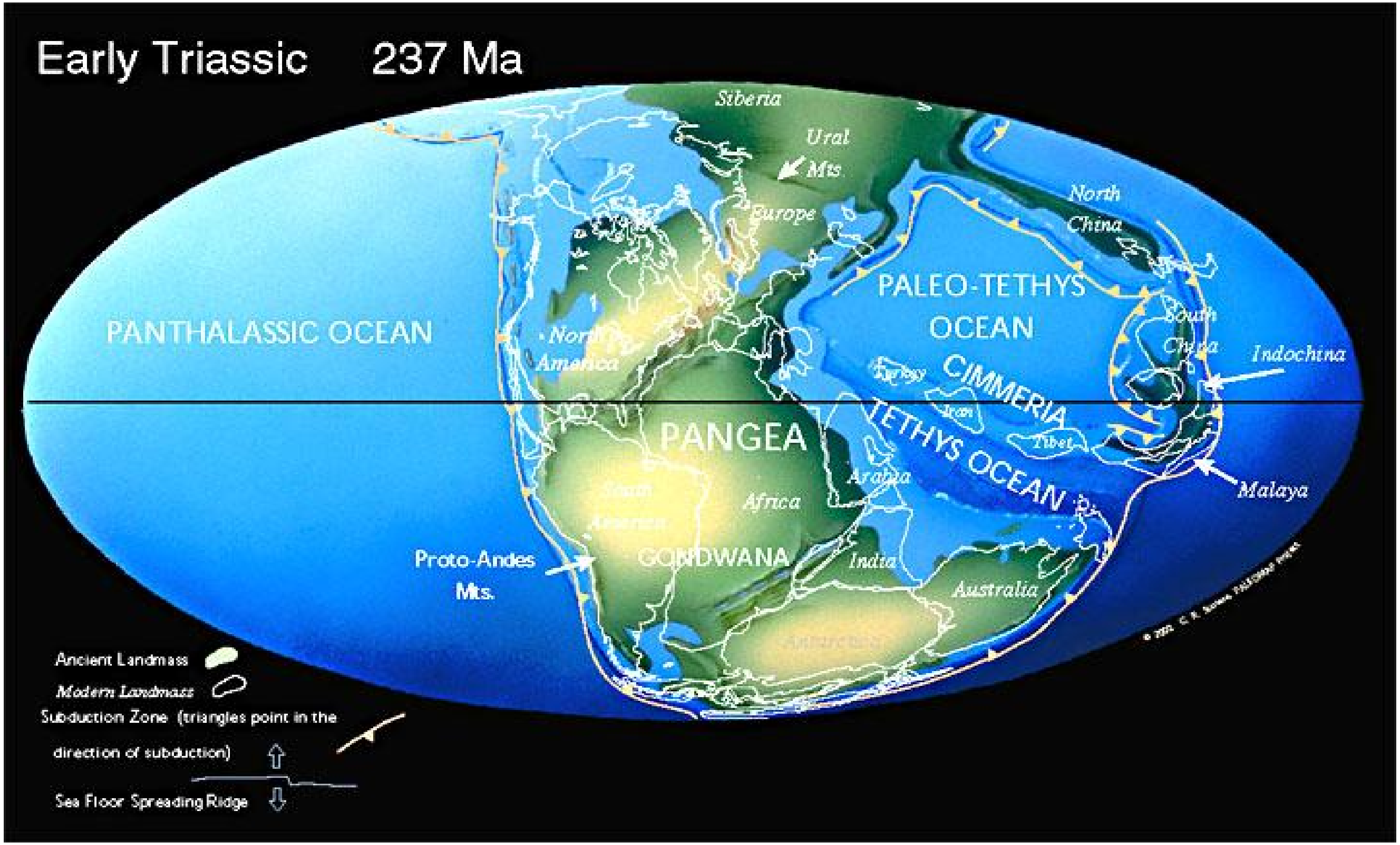
Huge Time Scale

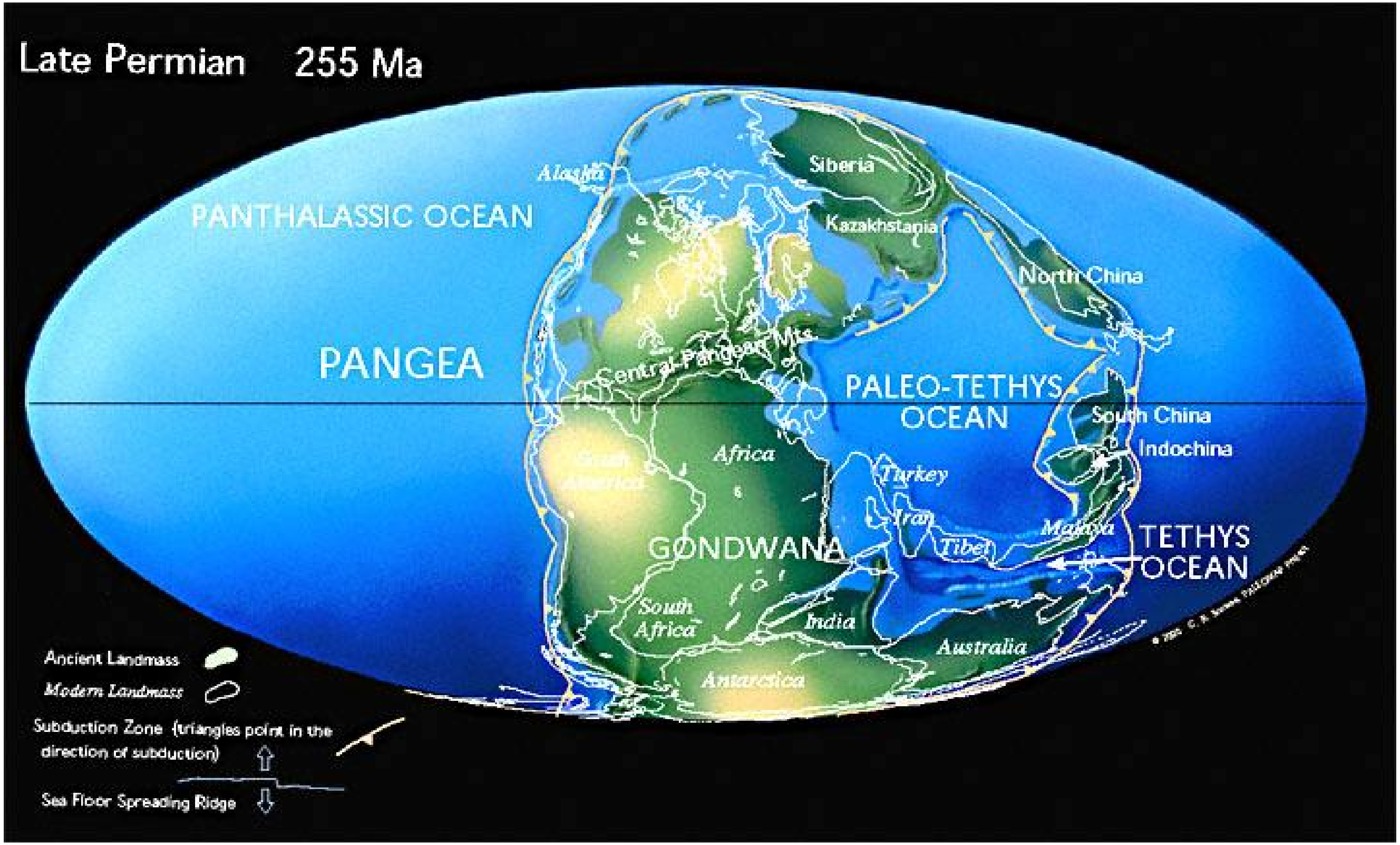


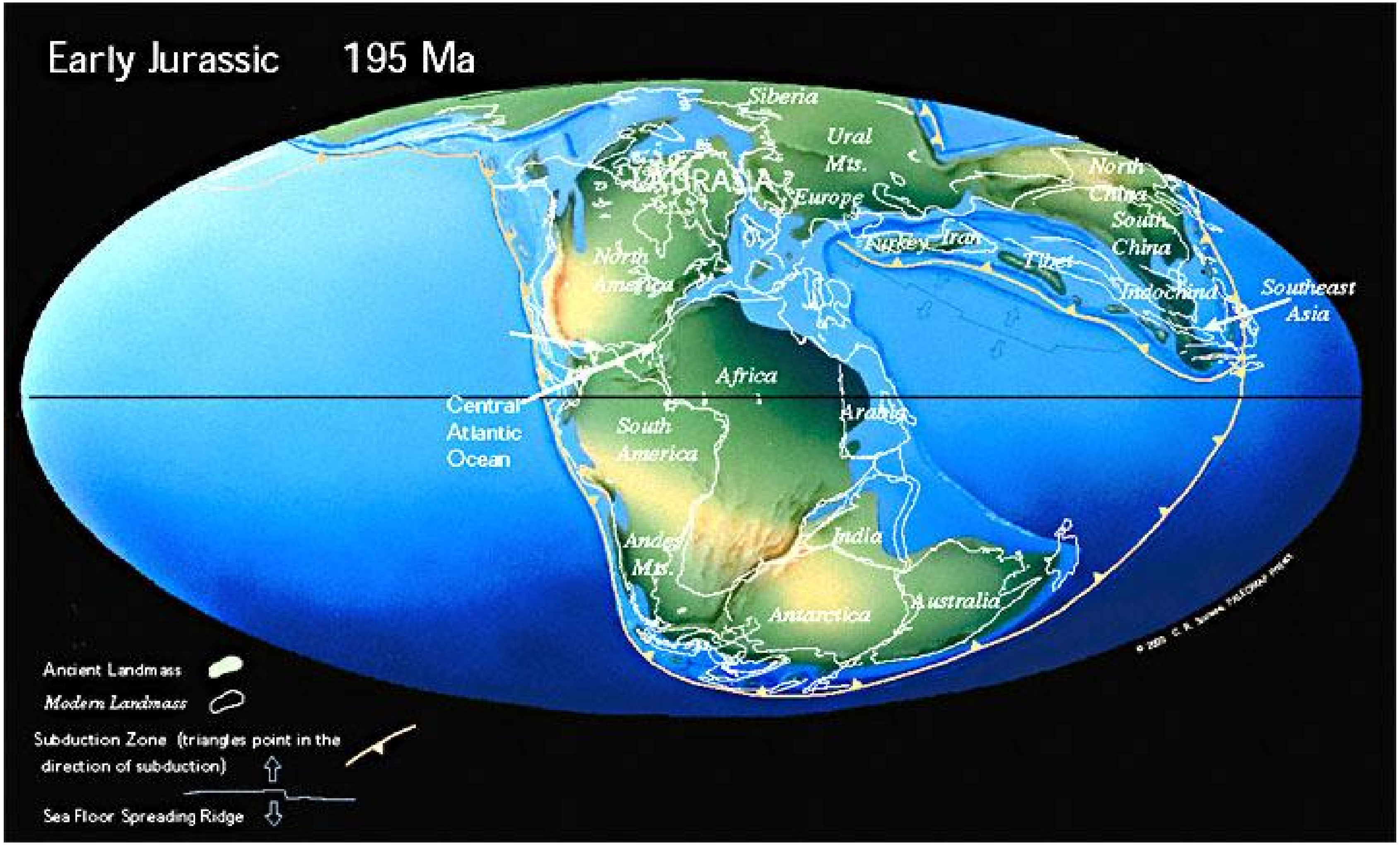
Early Carboniferous 356 Ma
PANTHALASSIC OCEAN
Siberia
Ural Mts.
Kazakstania
North China
South China
PALEO-
TETHYS
OCEAN
EURAMERICA
Malaya
Variscan Mts.
Arabia
Africa
Australia
RHEIC OCEAN
India
South America
Antarctica
GONDWANA
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge

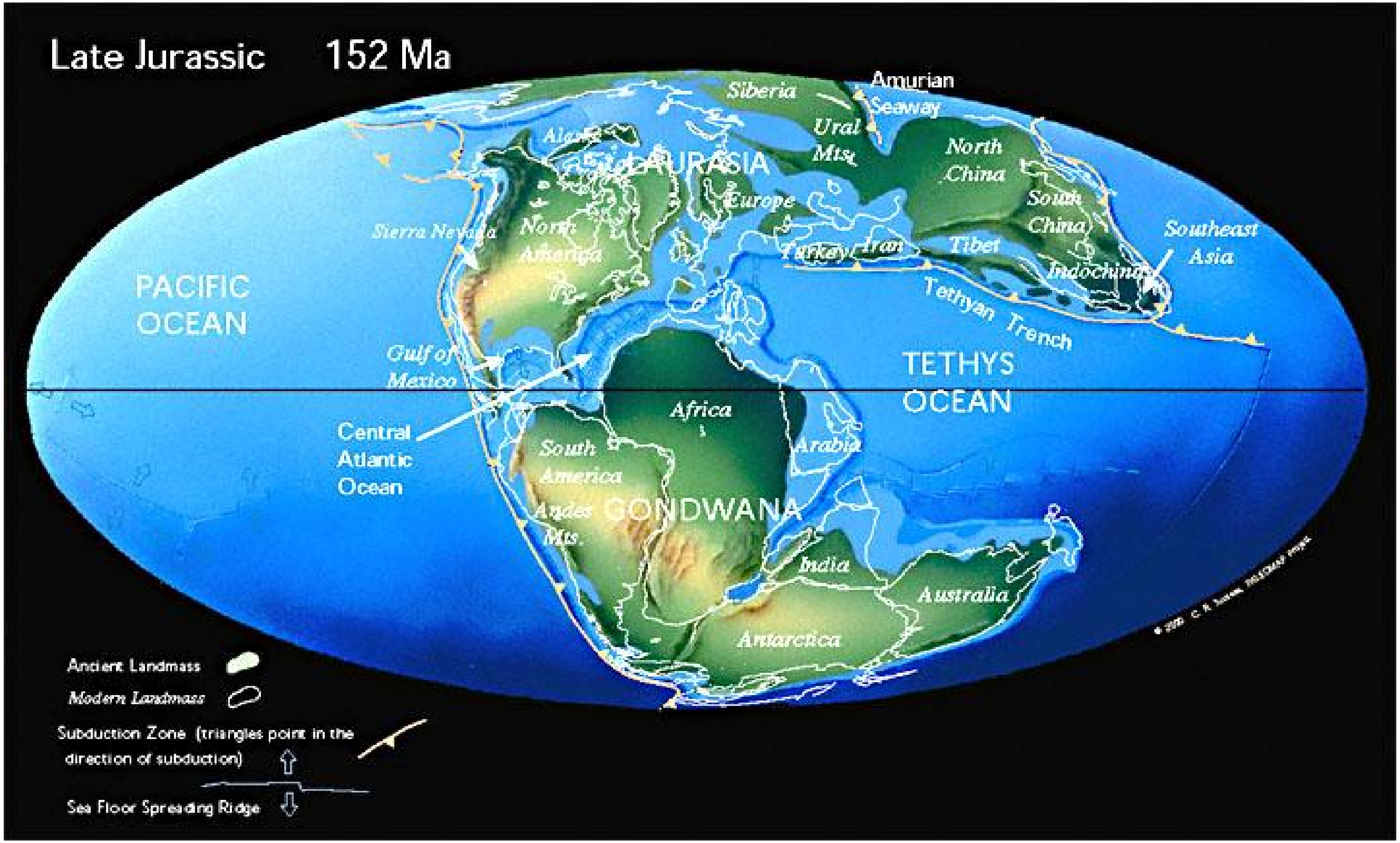


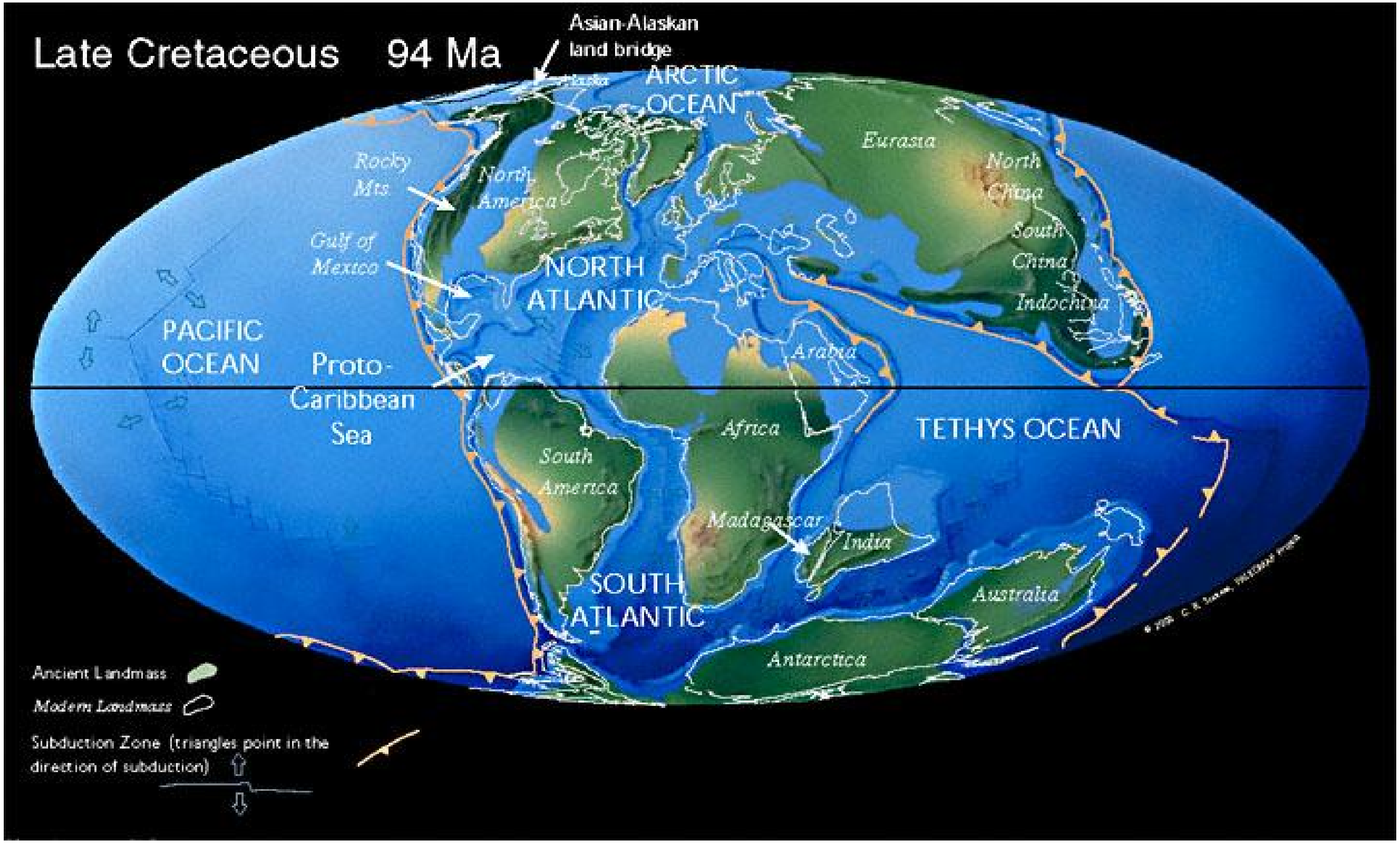
Late Carboniferous 306 Ma
PANTHALASSIC OCEAN
Siberia
Kazakstania
Ural Mts.
North China
PANGEA
PALEO-TETHYS SEA
Ancestral Rockies
Ouachita Mts.
Meseta
South China
South America
Africa
GONDWANA
India
Australia
Antarctica
Madagascar
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge

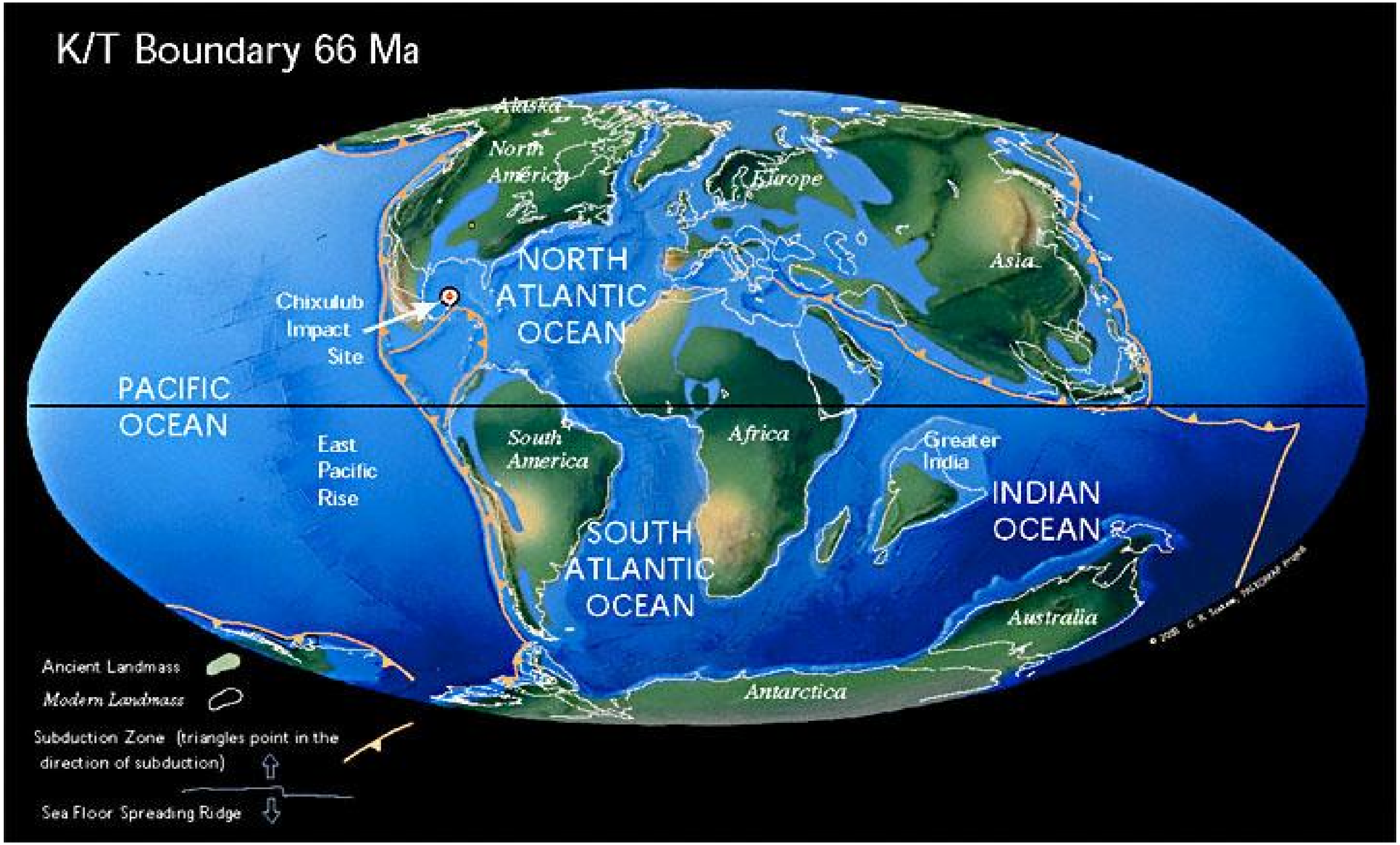

Early Triassic 237 Ma
Siberia
Ural Mts.
Europe
North China
PANTHALASSIC OCEAN
PALEO-TETHYS OCEAN
South China
Indochina
North America
CIMMERIA
Turkey
Iran
Tibet
TETHYS OCEAN
PANGEA
Arabia
Malaya
South America
Africa
GONDWANA
India
Australia
Proto-Andes Mts.
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge

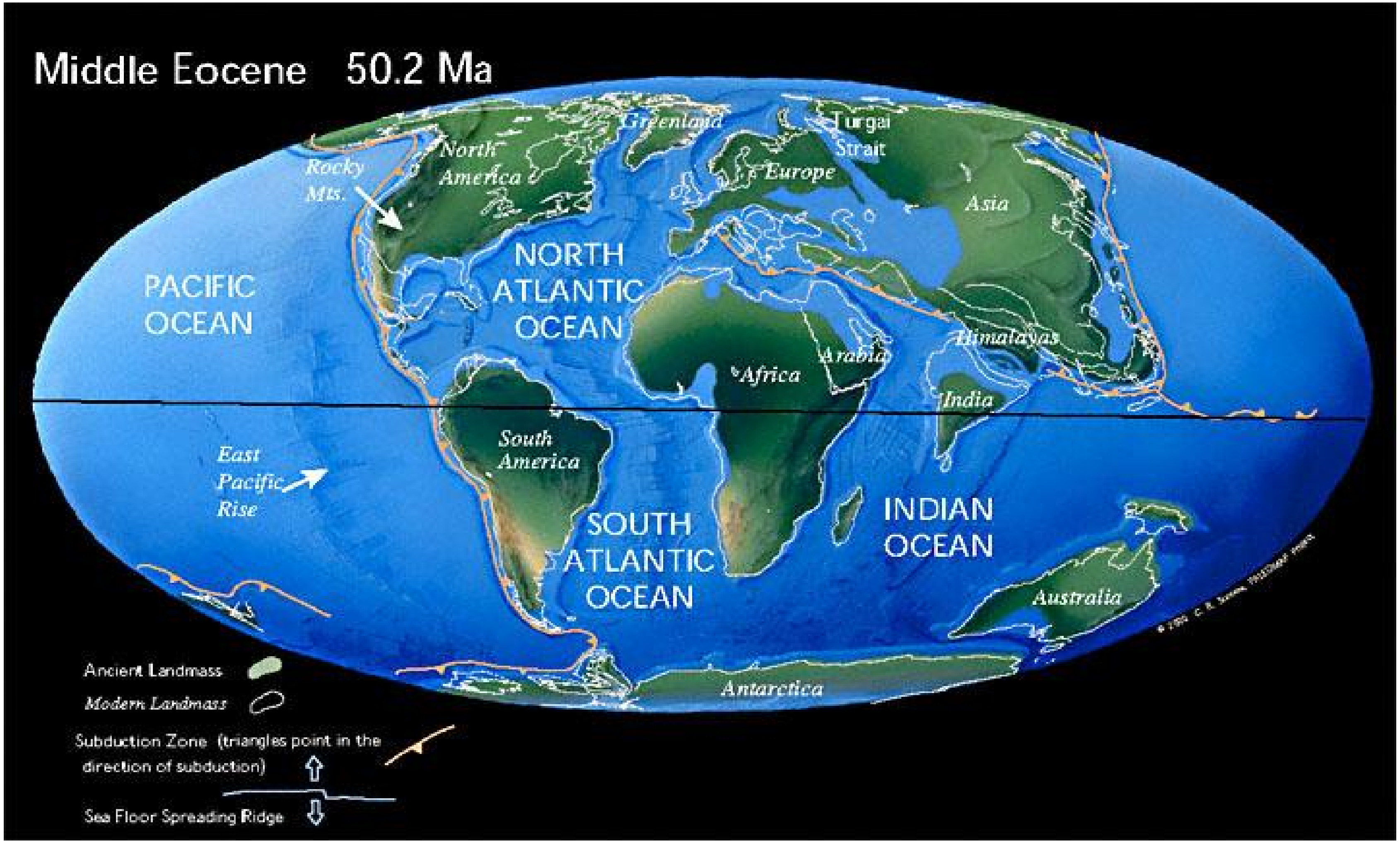

Late Permian 255 Ma
PANTHALASSIC OCEAN
PANGEA
Alaska
Siberia
Kazakhstania
North China
Central Pangean Mts.
PALEO-TETHYS OCEAN
South China
Indochina
Africa
South America
Turkey
Iran
GONDWANA
Tibet
Malaya
TETHYS OCEAN
South Africa
India
Australia
Antarctica
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge

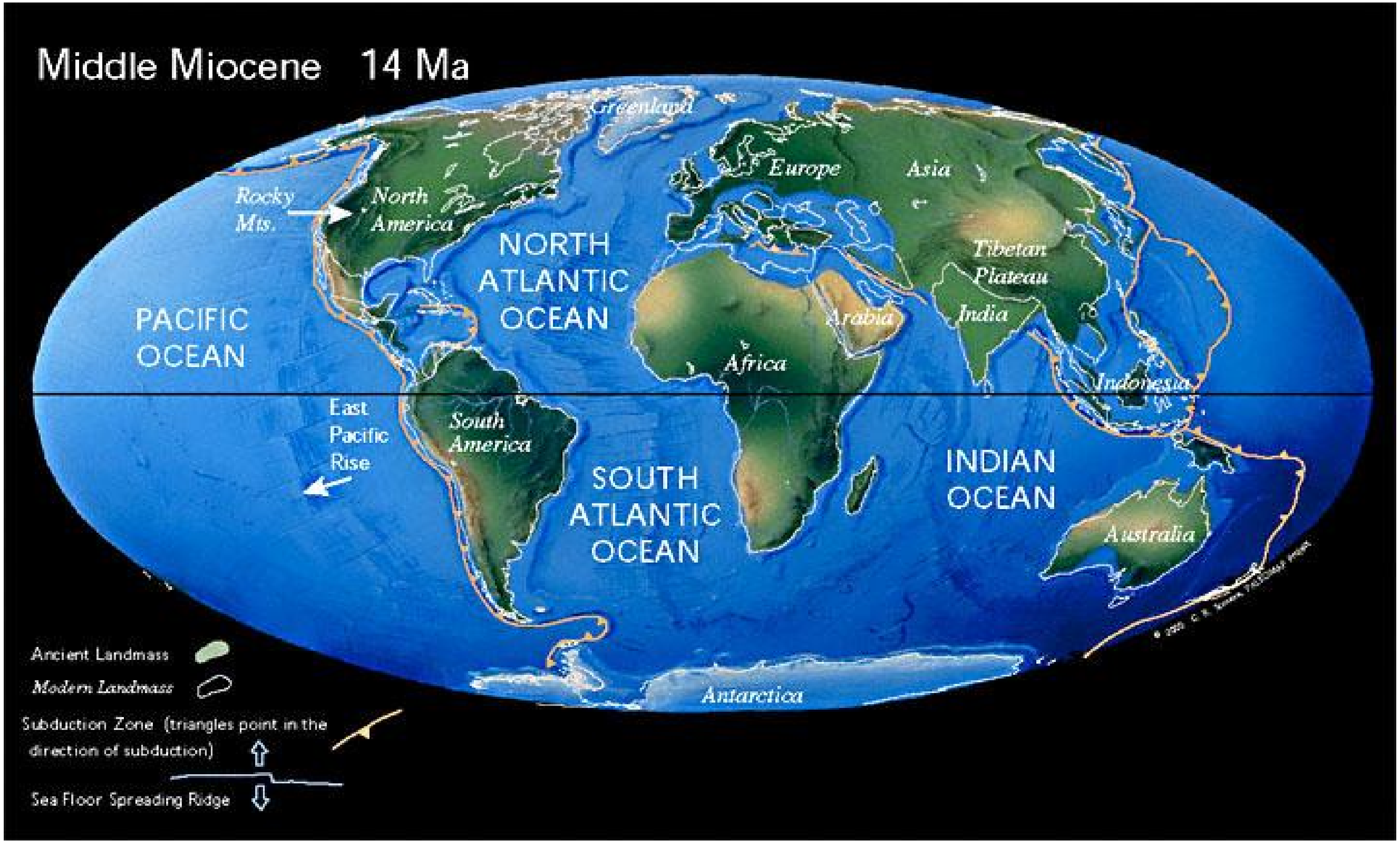

Early Jurassic 195 Ma
Siberia
Ural Mts.
LAURASIA
Europe
North China
South China
Turkey
Iran
Tibet
Indochina
Southeast Asia
North America
Africa
Central Atlantic Ocean
South America
Arabia
India
Andes Mts.
Antarctica
Australia
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge


Late Jurassic 152 Ma
Siberia
Amurian Seaway
Ural Mts.
LAURASIA
North China
South China
Europe
Sierra Nevada
North America
Southeast Asia
Turkey
Iran
Tibet
Indochina
PACIFIC OCEAN
Tethyan Trench
Gulf of Mexico
TETHYS OCEAN
Africa
Central Atlantic Ocean
South America
Arabia
Andes Mts.
GONDWANA
India
Australia
Antarctica
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge


Late Cretaceous 94 Ma
Asian-Alaskan
land bridge
ARCTIC
OCEAN
Eurasia
North
China
South
China
Indochina
Rocky
Mts.
North
America
Gulf of
Mexico
NORTH
ATLANTIC
PACIFIC
OCEAN
Proto-
Caribbean
Sea
Arabia
Africa
TETHYS OCEAN
South
America
Madagascar
India
SOUTH
ATLANTIC
Australia
Antarctica
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the
direction of subduction)


K/T Boundary 66 Ma
Alaska
North America
Europe
Asia
NORTH ATLANTIC OCEAN
Chixulub Impact Site
PACIFIC OCEAN
East Pacific Rise
South America
Africa
Greater India
INDIAN OCEAN
SOUTH ATLANTIC OCEAN
Australia
Antarctica
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge


Middle Eocene 50.2 Ma
Greenland
Turgai Strait
North America
Rocky Mts.
Europe
Asia
NORTH ATLANTIC OCEAN
PACIFIC OCEAN
Himalayas
Arabia
Africa
India
South America
East Pacific Rise
SOUTH ATLANTIC OCEAN
INDIAN OCEAN
Australia
Antarctica
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge


Middle Miocene 14 Ma
Greenland
Europe
Asia
Rocky Mts.
North America
NORTH ATLANTIC OCEAN
Tibetan Plateau
PACIFIC OCEAN
Arabia
India
Africa
Indonesia
East Pacific Rise
South America
SOUTH ATLANTIC OCEAN
INDIAN OCEAN
Australia
Antarctica
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
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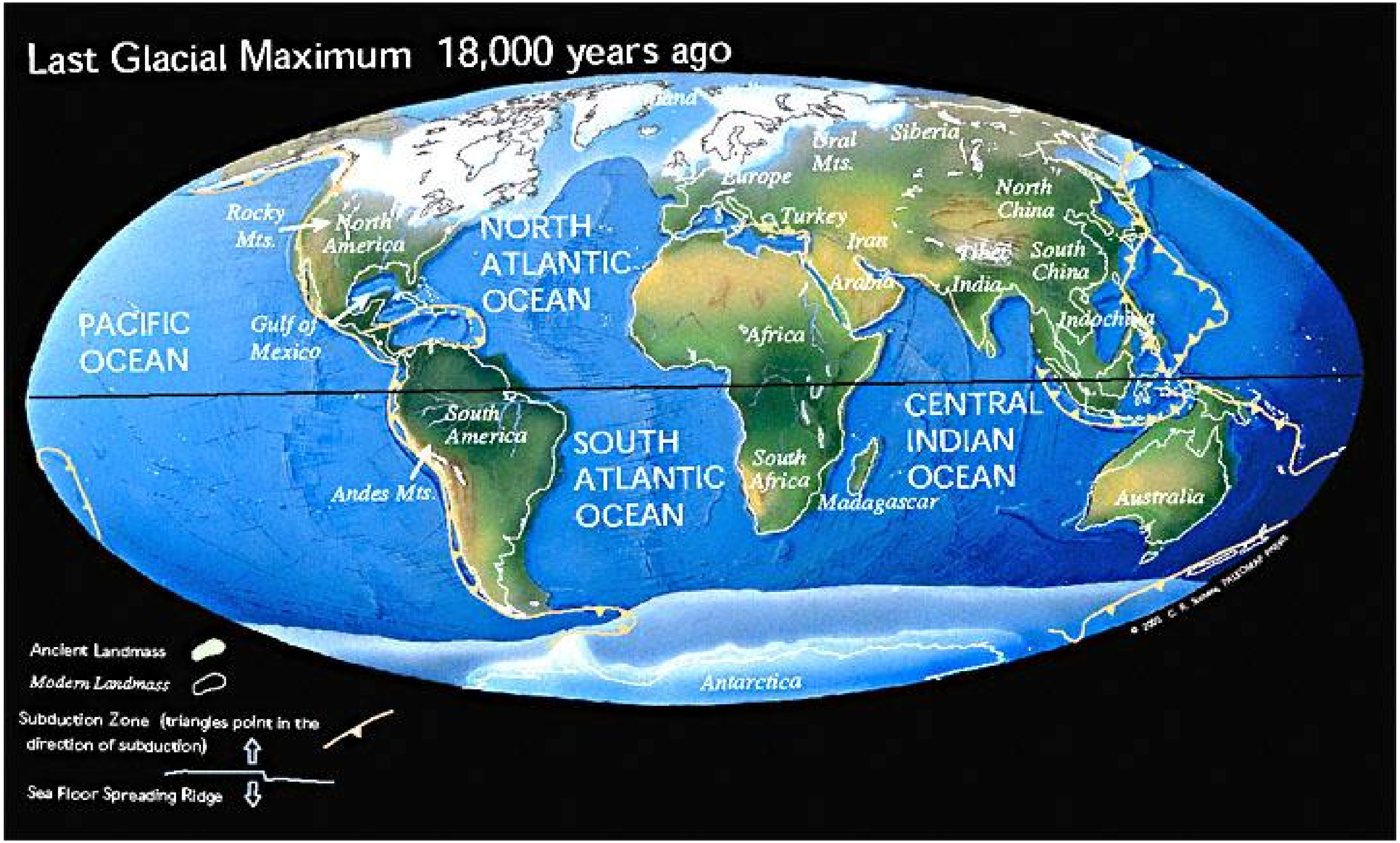
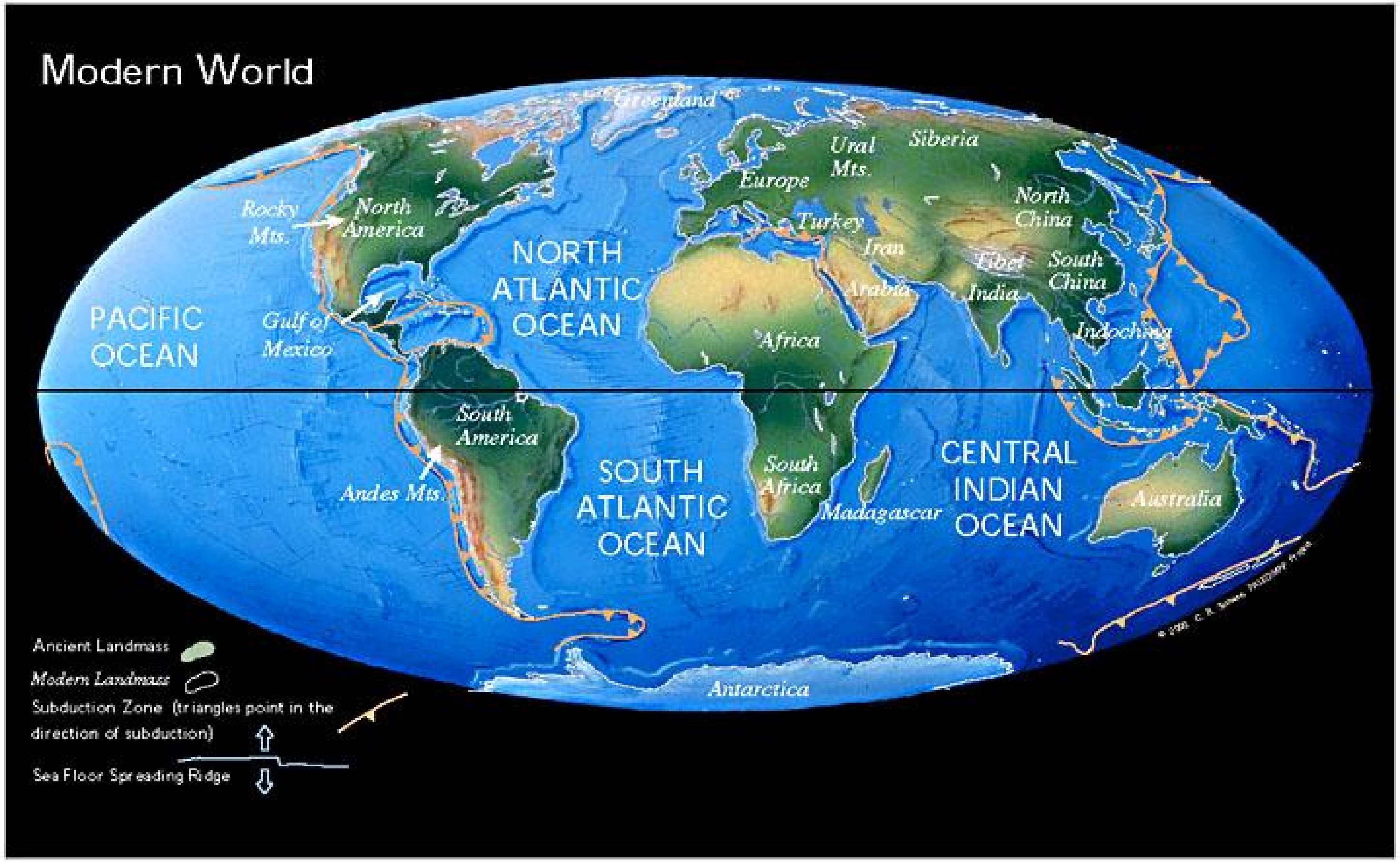
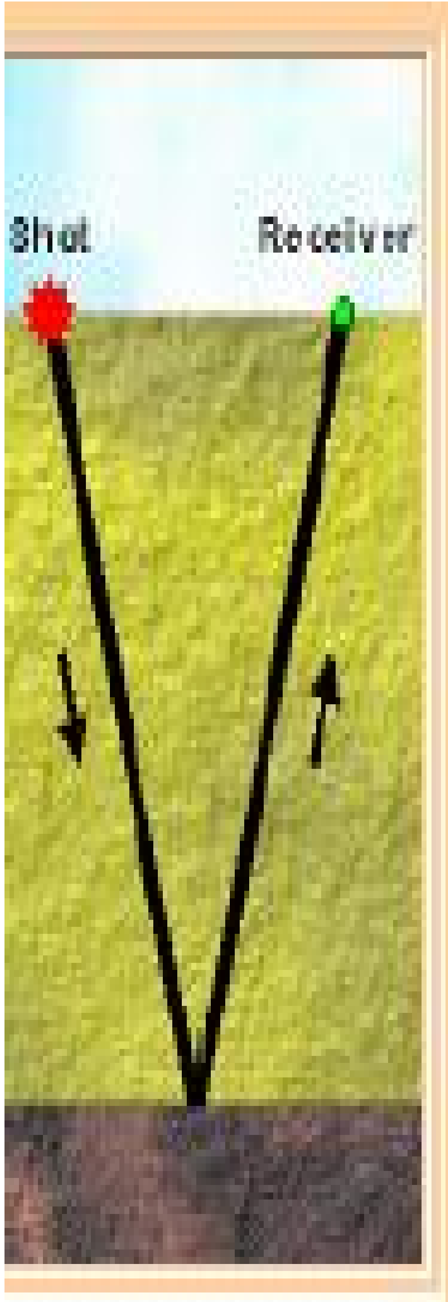
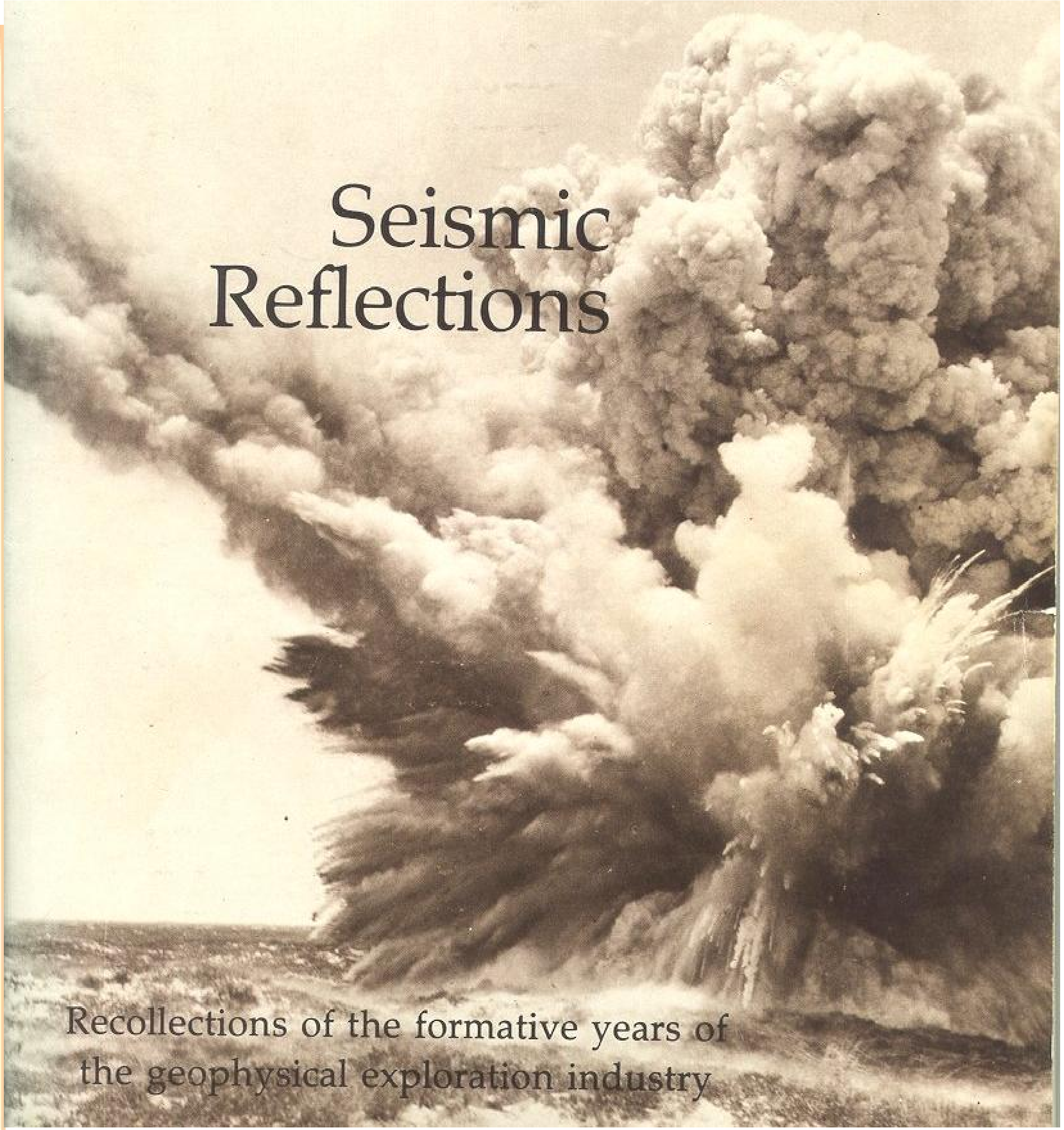

Last Glacial Maximum 18,000 years ago
Siberia
Ural Mts.
Europe
North China
Rocky Mts.
North America
NORTH ATLANTIC OCEAN
Turkey
Iran
Tibet
South China
Arabia
India
PACIFIC OCEAN
Gulf of Mexico
Africa
Indochina
CENTRAL INDIAN OCEAN
South America
SOUTH ATLANTIC OCEAN
South Africa
Andes Mts.
Madagascar
Australia
Antarctica
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge


Modern World
Greenland
Siberia
Ural Mts.
Europe
North China
Rocky Mts.
North America
Turkey
Iran
Tibet
South China
India
Arabia
NORTH ATLANTIC OCEAN
PACIFIC OCEAN
Gulf of Mexico
Africa
Indochina
South America
Andes Mts.
SOUTH ATLANTIC OCEAN
South Africa
Madagascar
CENTRAL INDIAN OCEAN
Australia
Antarctica
Ancient Landmass
Modern Landmass
Subduction Zone (triangles point in the direction of subduction)
Sea Floor Spreading Ridge

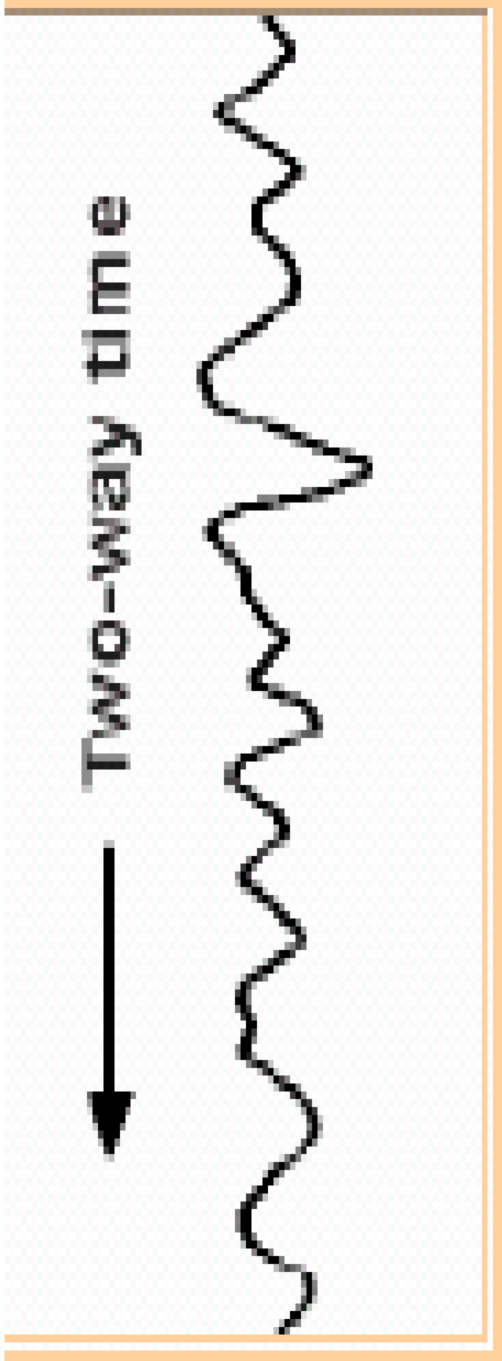
Geophysics





A refraction shot in West Texas. 2,000 pounds of dynamite shot by Humble Oil & Refining Company (now Exxon), 1930.

Modern technology



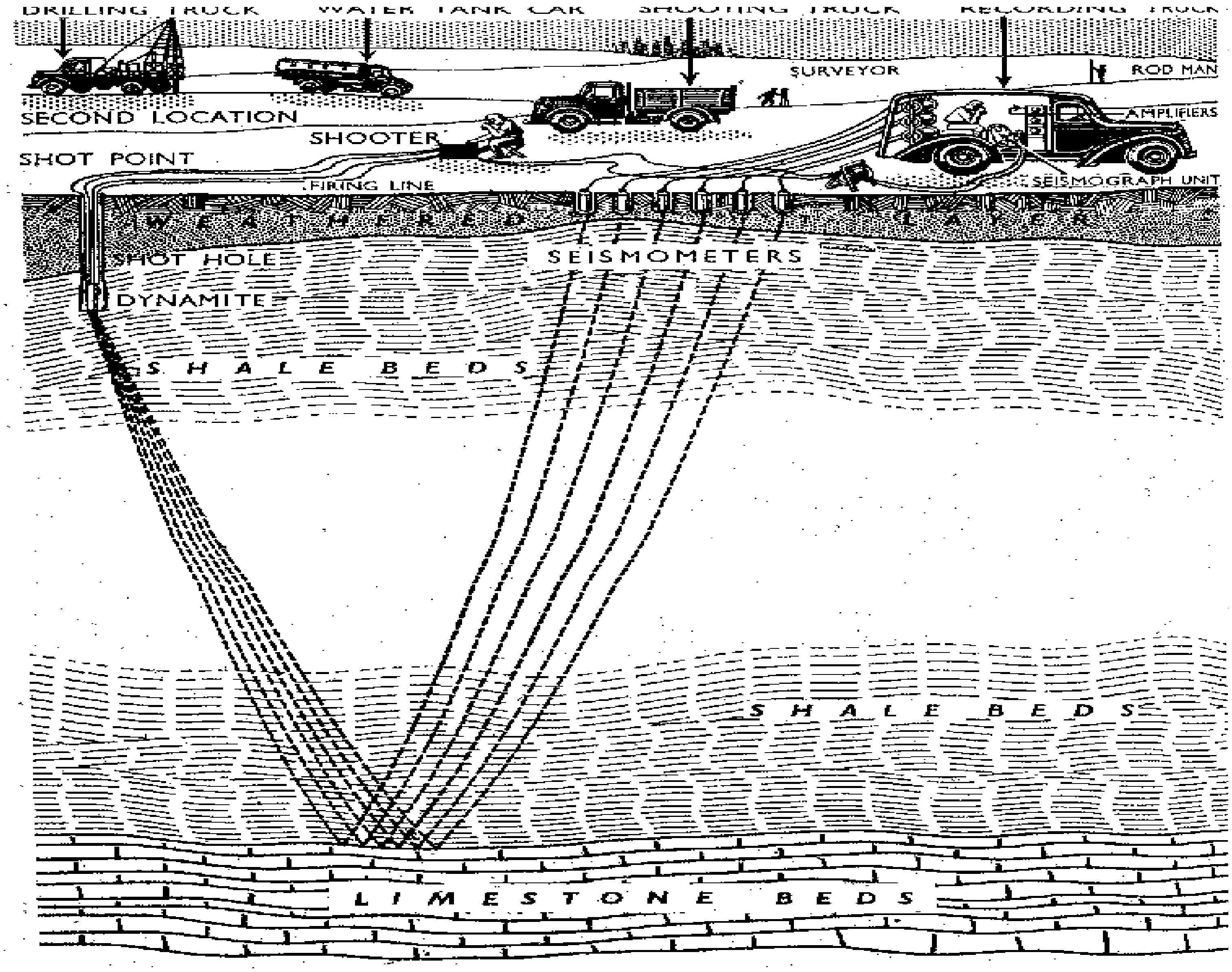
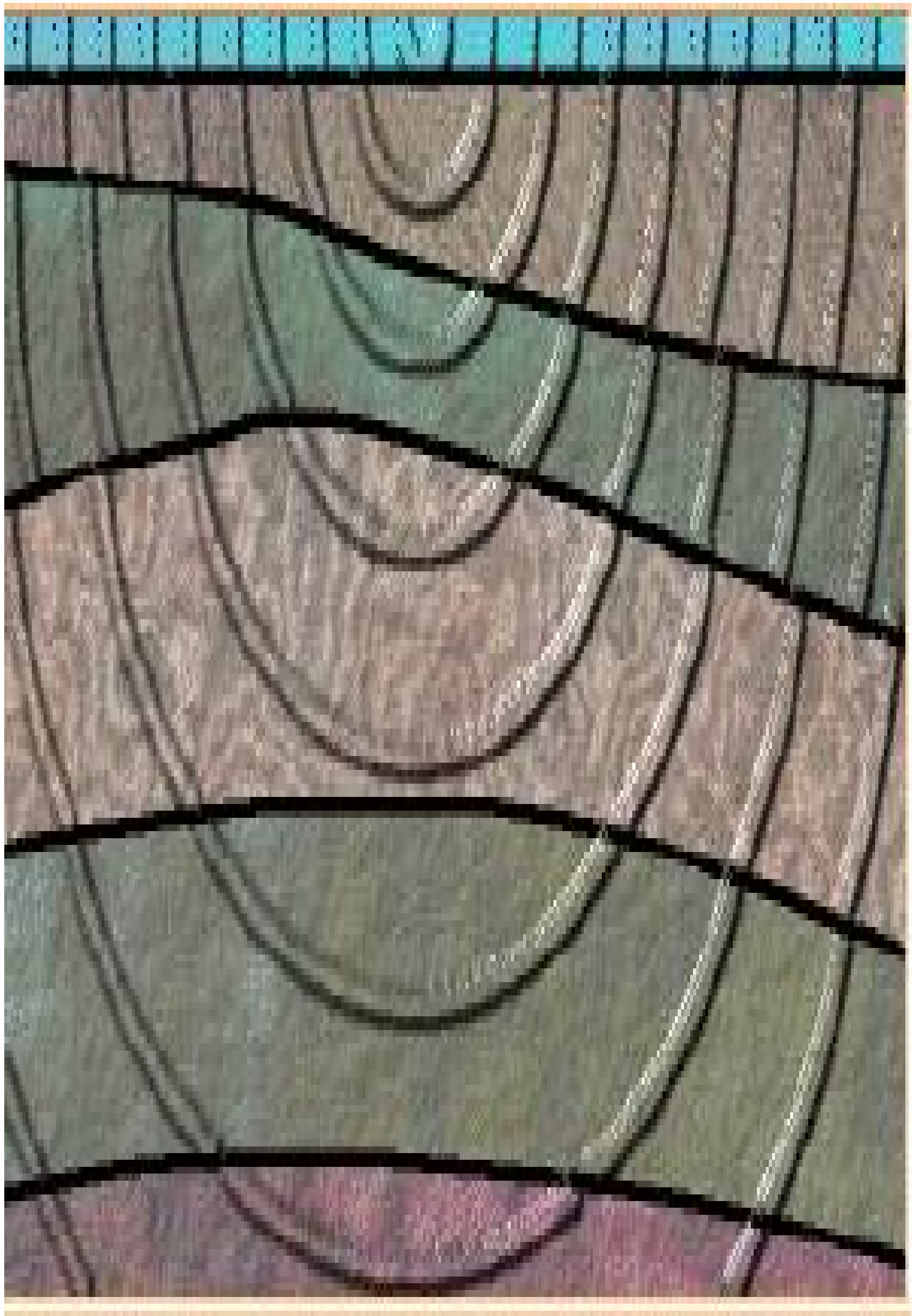
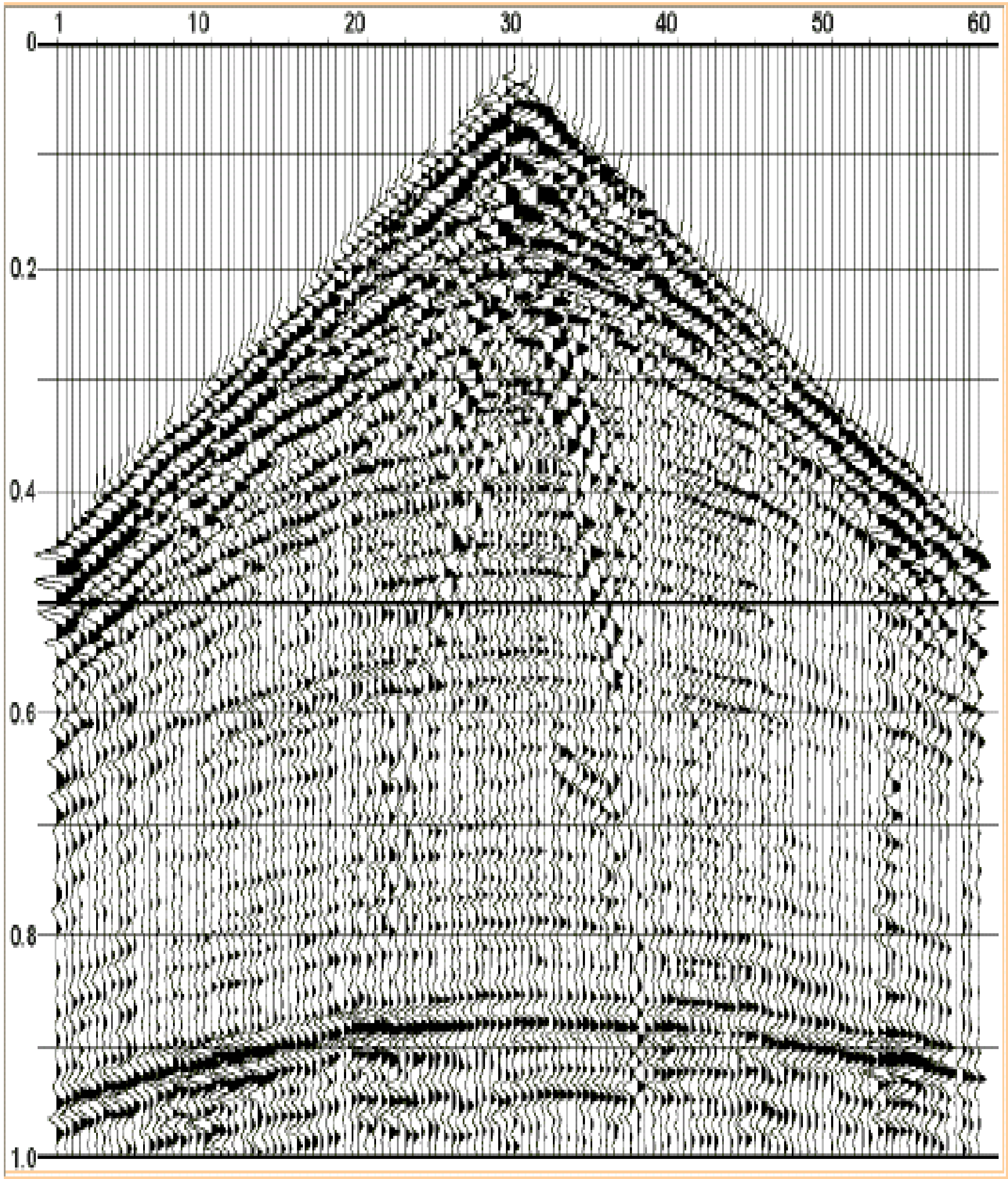


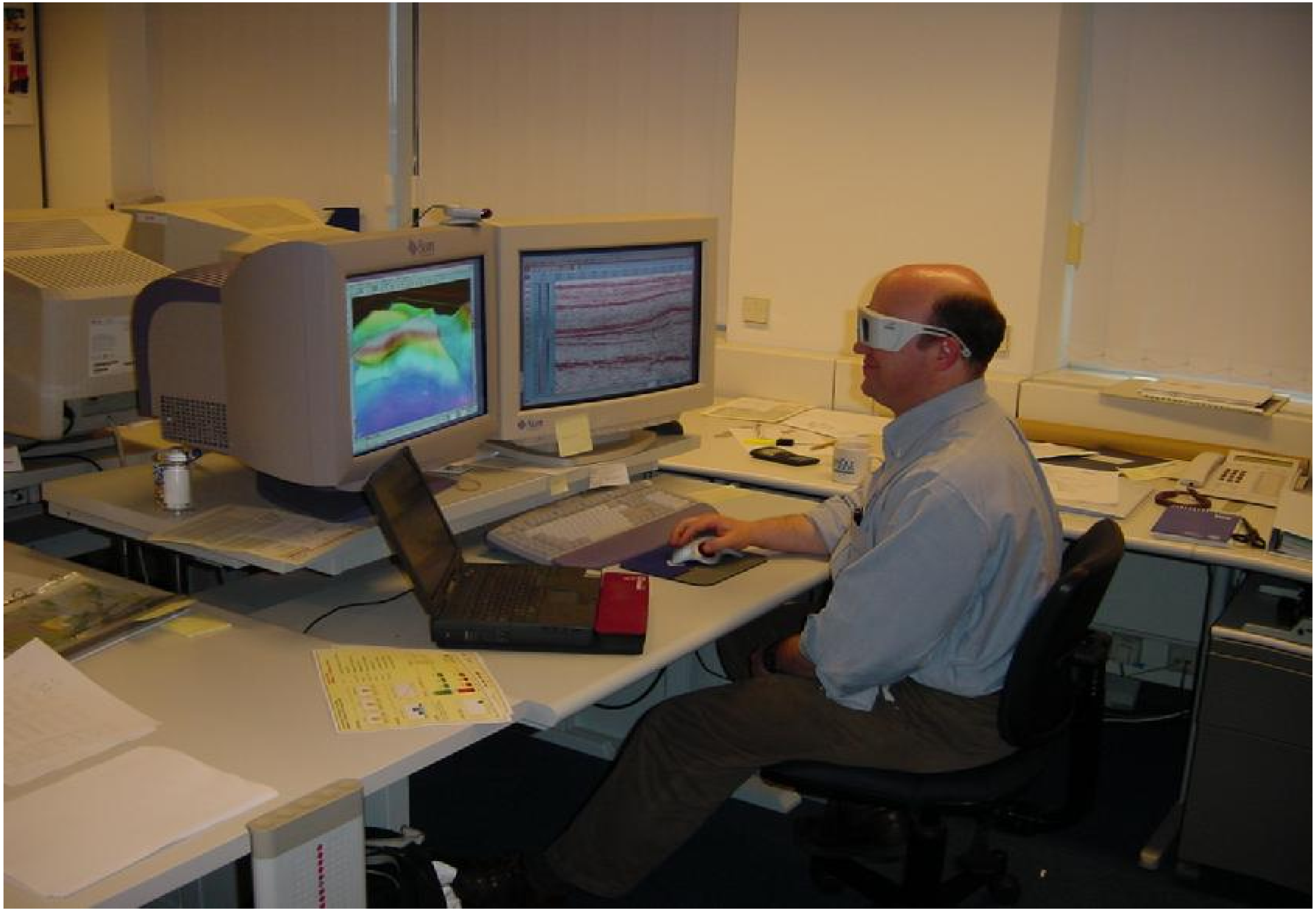
Fig. 14. Diagrammatic scheme for Reflection Shooting.

Petroleum Handbook 1948 (Shell)

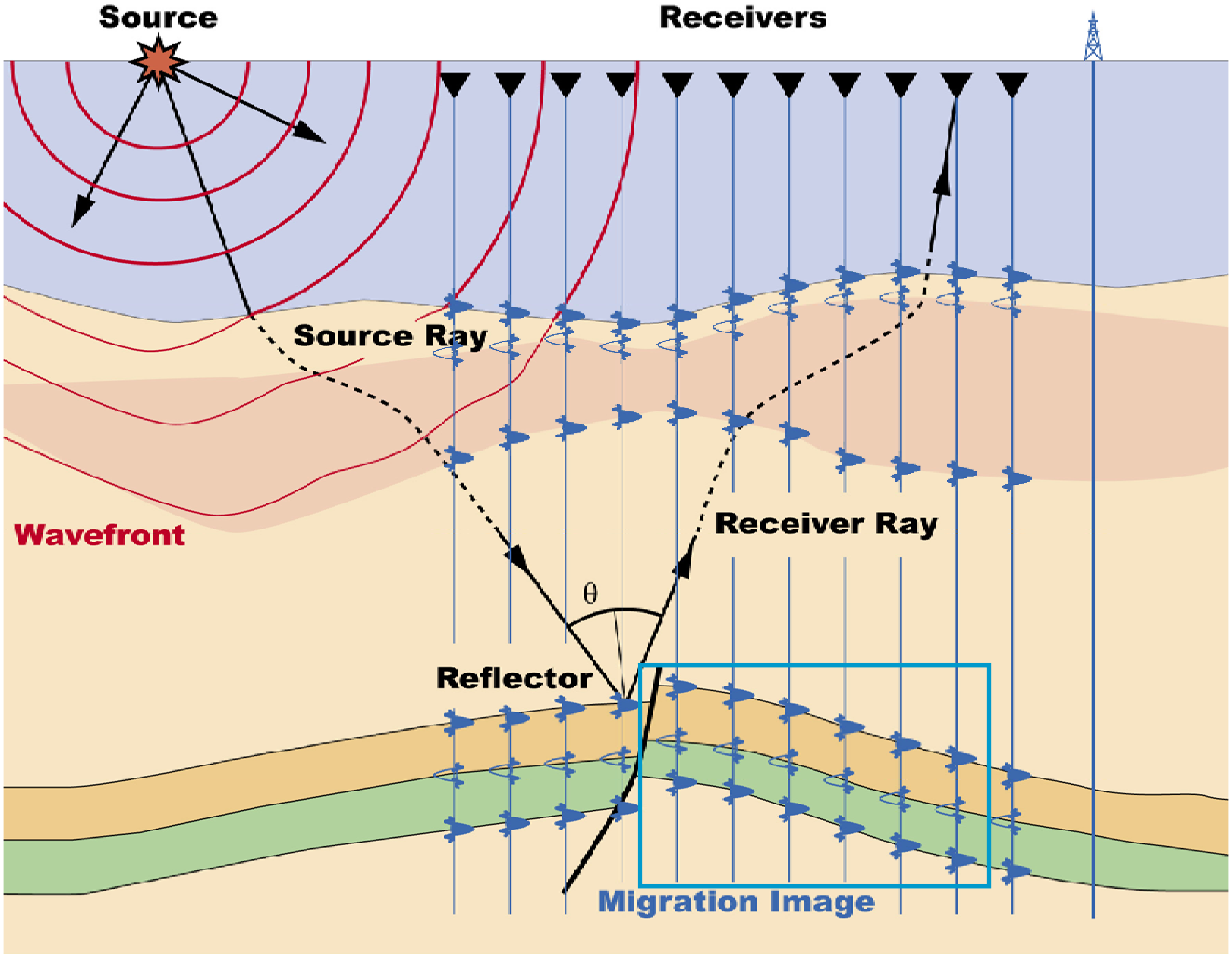



1
10
20
30
40
50
60
0
0.2
0.4
0.6
0.8
1.0

The Interpreter at work



(Jaap van der Toorn, NAM TGS-S)


Source
Receivers
Source Ray
Wavefront
Receiver Ray
θ
Reflector
Migration Image

wave equation

$$\Omega v = f$$

source

pressure field

wave equation operator

$$\Omega = \left(\frac{1}{c^2} \partial_{tt} - \Delta \right)$$

Laplacian

c is sound velocity; c = c(x,y,z)

specification *c* for different layers is velocity model *m*

$$\Omega = \Omega(m)$$

Inversion

Find the best model *m* that explains the data

$$\min_{m} J(m)$$ **with**

$$J(m) = \sum_{l} (v_l(m) - v_l^{\text{obs}})^2$$

$$s.t., \Omega(m)v = f$$

General approach too difficult, *J* has local minima

Migration: **an initial model, m_0, is assumed known**

A large number of approaches for the Migration problem

Here just one example

wave equation | Helmholtz equation

$$\left(\frac{1}{c^2}\partial_{tt} - \Delta\right)v = f \xrightarrow{\text{Fourier in time}} -\left(\frac{w^2}{c^2} + \Delta\right)\hat{v} = \hat{f}$$

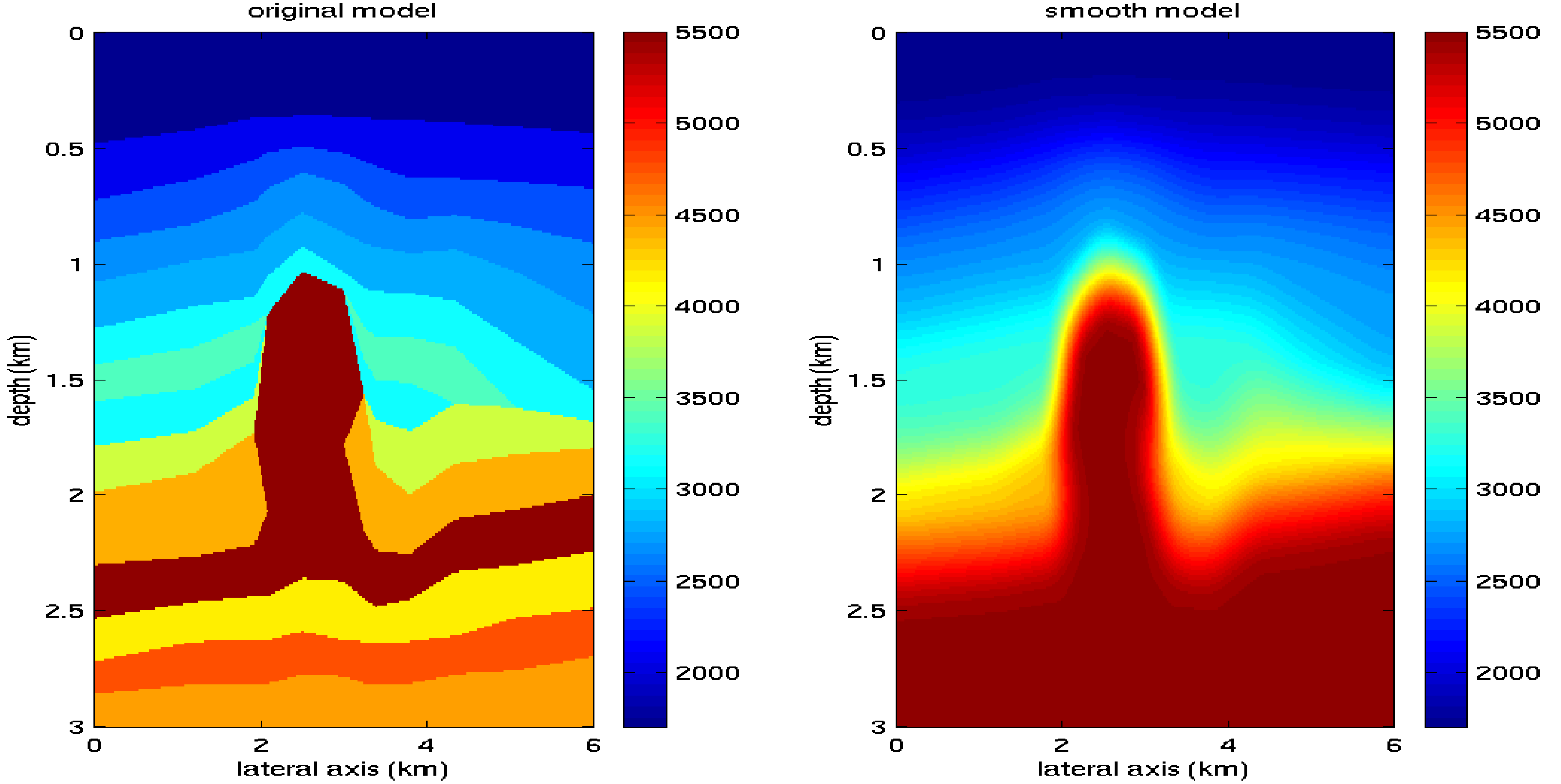
Discretize: $\boldsymbol{A}\,\boldsymbol{v} = \boldsymbol{F}$

Solution: $\boldsymbol{A} = \boldsymbol{L}\,\boldsymbol{U}$

$\boldsymbol{L}$ is lower triangular, $\boldsymbol{U}$ is upper triangular

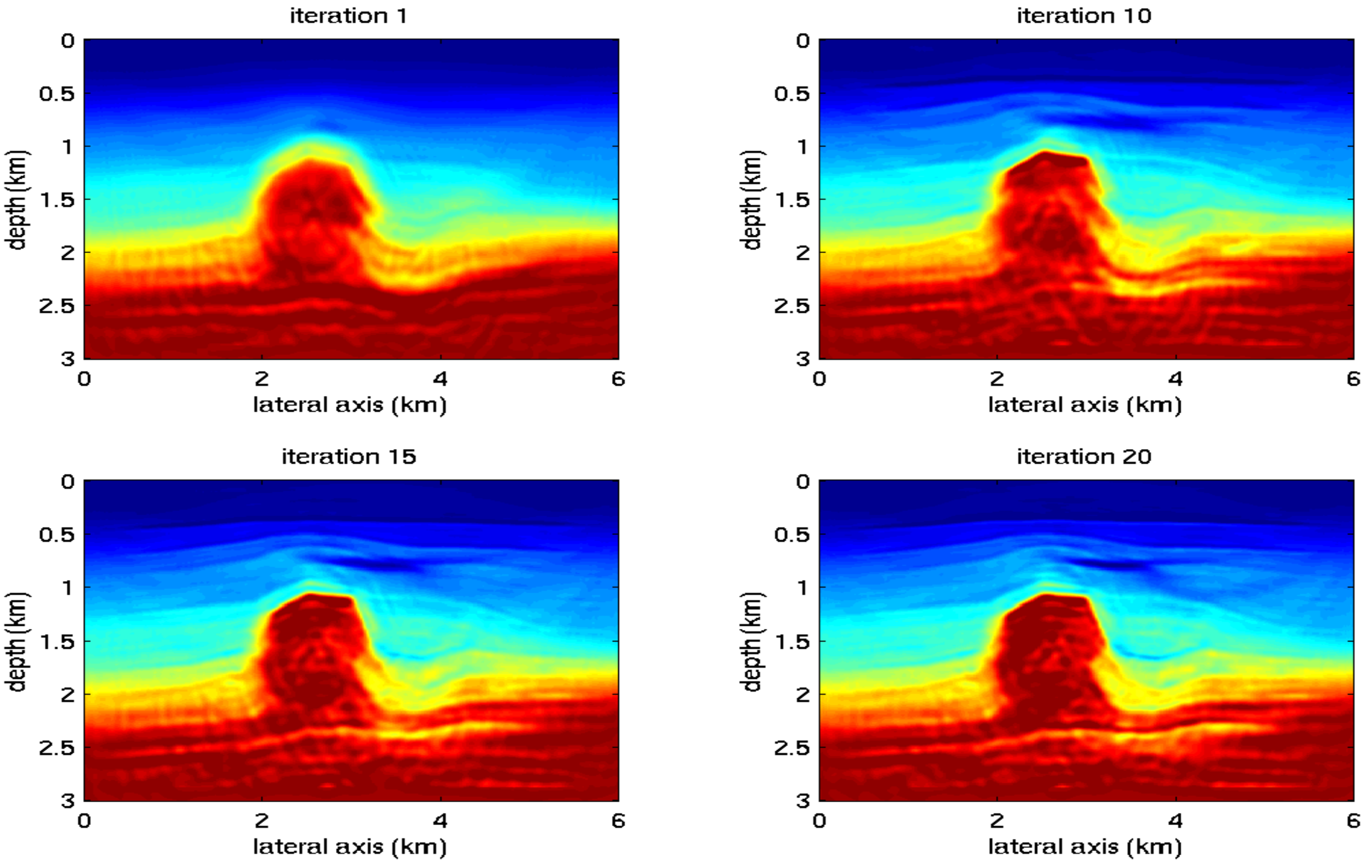
back substitution: $\boldsymbol{v} = \boldsymbol{U}^{-1}\boldsymbol{L}^{-1}\boldsymbol{F}$

2D example



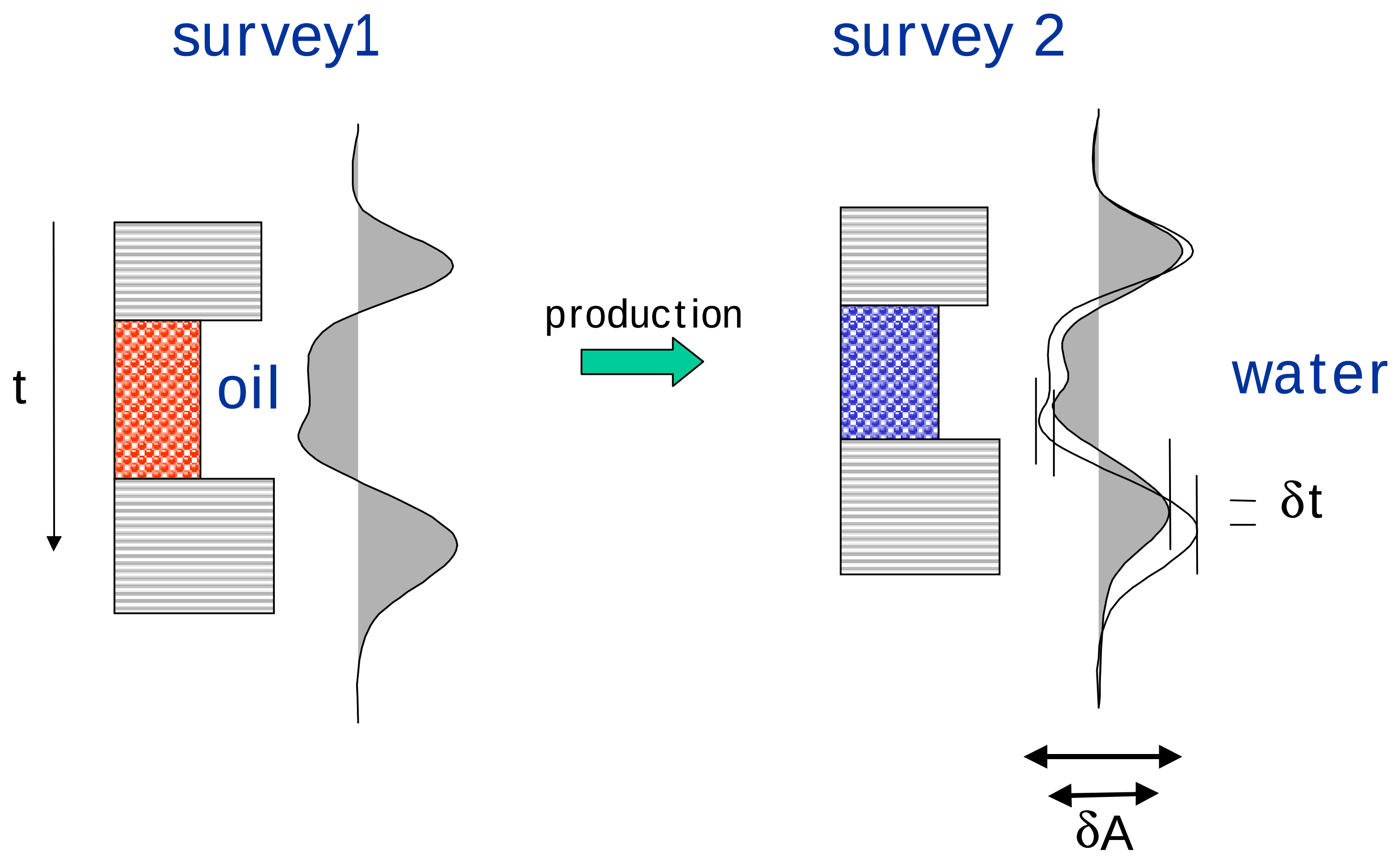
Original and smoothed model used for migration

2D example



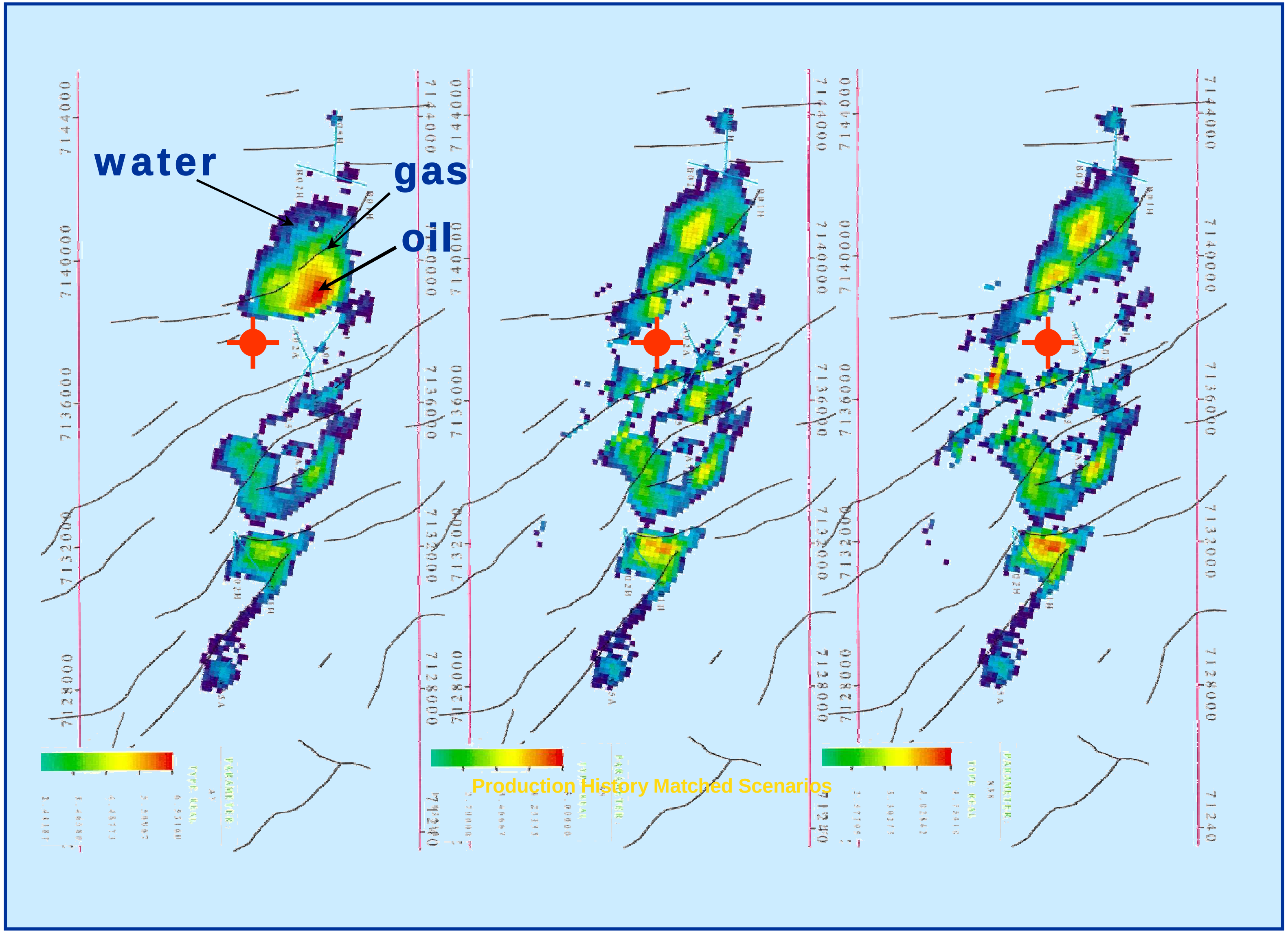
Iterative migration on smoothed model
(1,10,15, and 20 iterations)

The Geophysics of seismic 4D

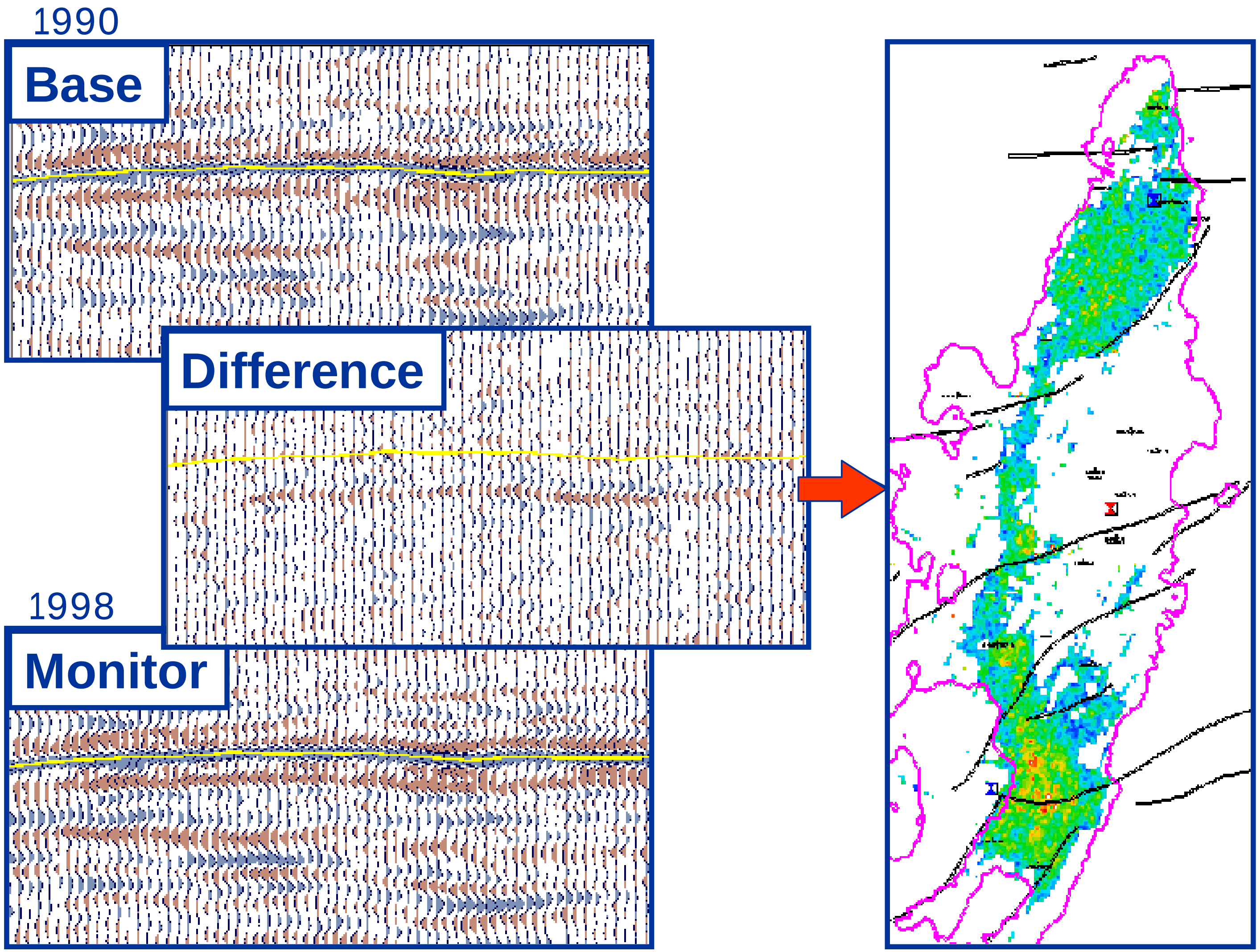


4D seismic amplitude and timing changes with production

3 of many history matched water flood simulations (simulation of production using water injection)



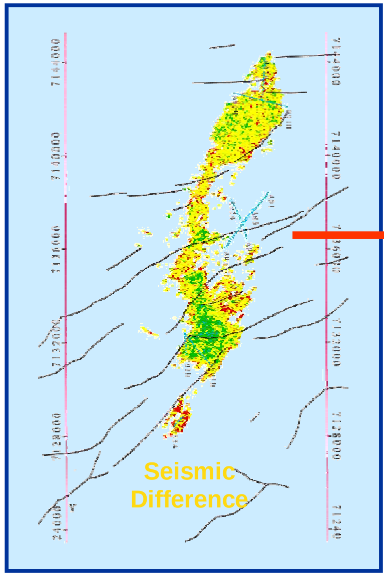
Time-lapse Seismic



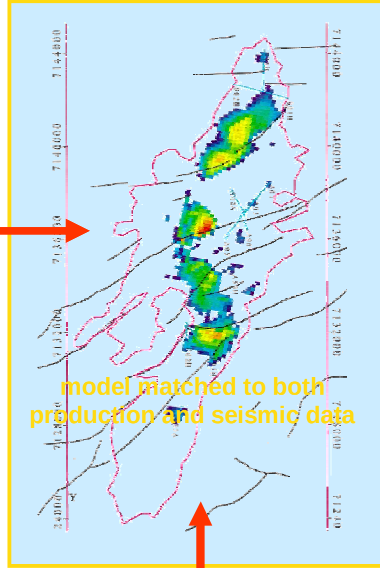
Dynamic model updating using 4D

Inputs

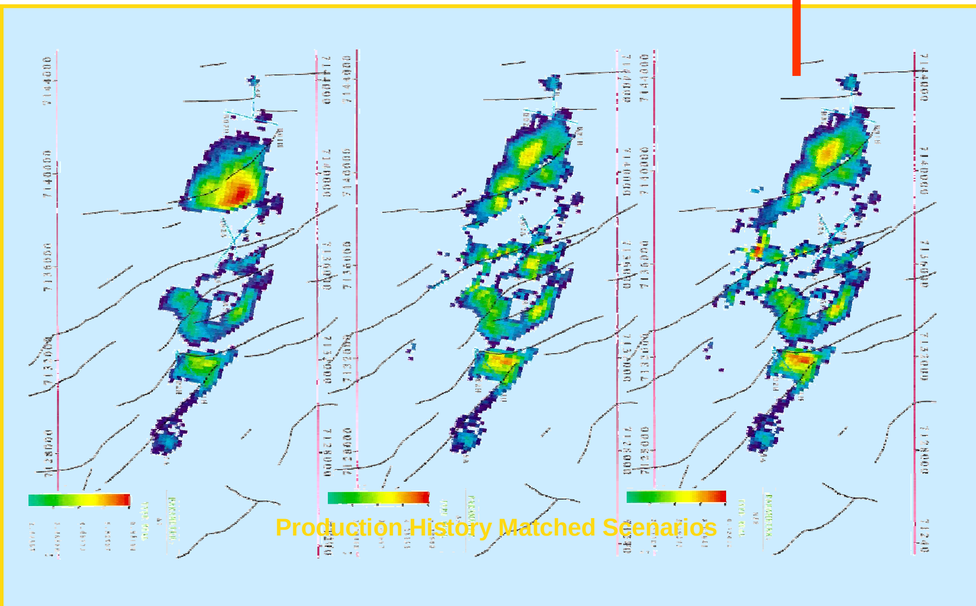
- 4D seismic



updated
flow model



- pre 4D
model
results



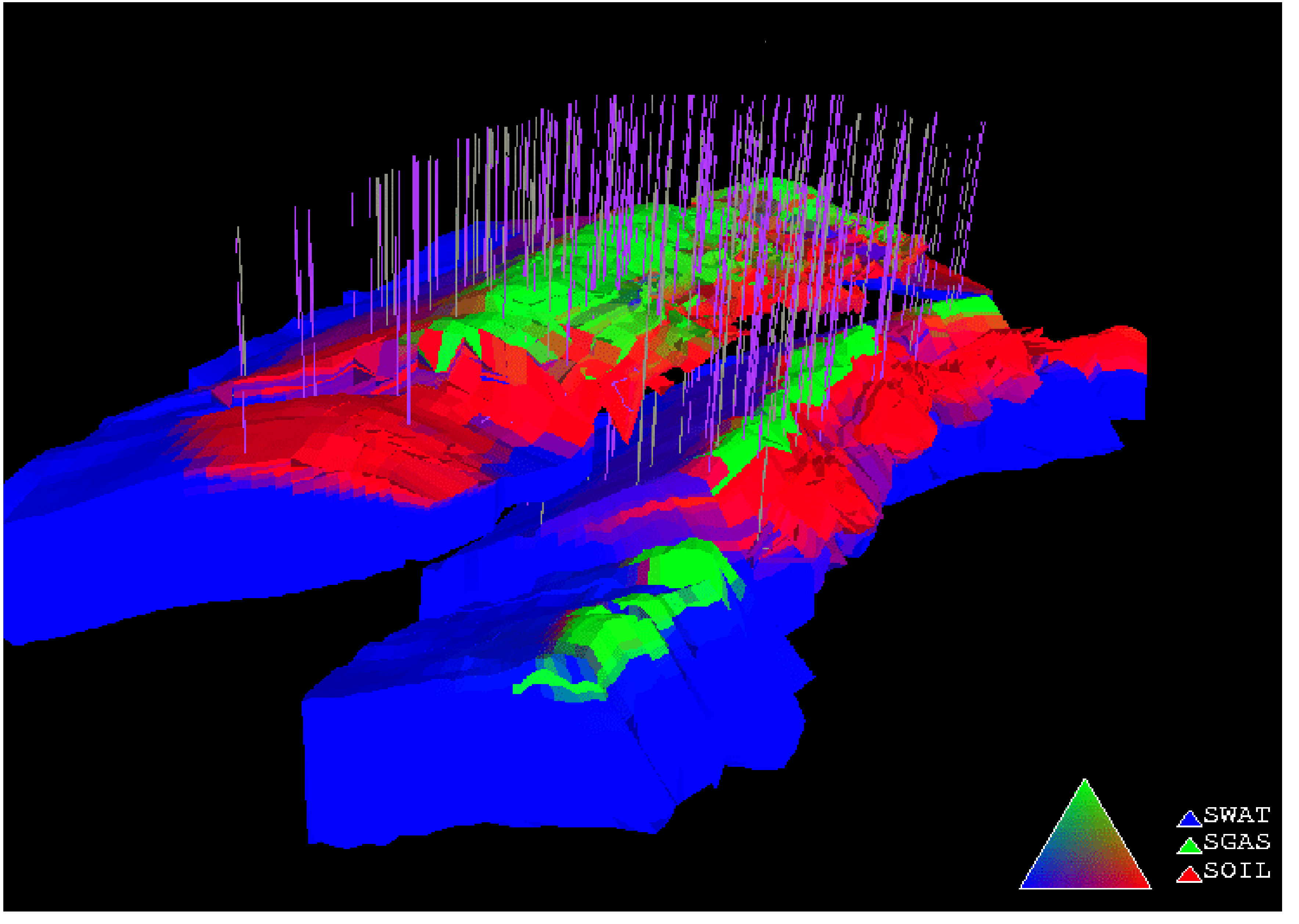
Flow Through Porous Media

What is Reservoir Engineering?

How to recover the maximum amount of oil (and/or gas) from a reservoir

- **Oil and gas under high pressure in sub-surface reservoirs**
- ***Inside*** **porous rock (e.g. sand stone) below a sealing cap rock**
- **Does more wells mean more oil?**
  - **Pressure drops below reservoir pressure, oil flow stops**
  - **Improve recovery by water injection (or gas): pump up the pressure**
  - **Or more advanced methods: steam injection, surfactants**
- **Where to drill these wells and how and when**
  - **vertical wells, deviated or horizontal wells**
  - **multi-laterals (more expensive)**
- **Reservoir engineers design a “Field development plan”**

The Brent oil field in the North See



To optimize oil and gas recovery computer simulations are used

- **Capture reservoir structure and geometry in a discrete model**
- **Compute how oil and gas flows through the reservoir rock**

  Depends on

  - **initial state (pressure, which fluids etc.)**
  - **wells**
  - **Fluid properties(oil viscosity), rock properties (porosity, permeability)**

Fast method to solve the flow equations is necessary

- **Simulate the history of a reservoir over 20-50 years**
- **Huge models with 100,000 to 1,000,000 grid cels**

Flow Equations, to be solved numerically
Can be derived from *Navier Stokes Equation*

$$\frac{\partial}{\partial t} S = \partial \cdot u + Q$$

Saturation Equation

- source (injection)
- total flow
- divergence
- saturation

$$u_p = k_{rel} (K / m) \nabla (p + p_{cap} + r_p h g)$$

Darcy's Law

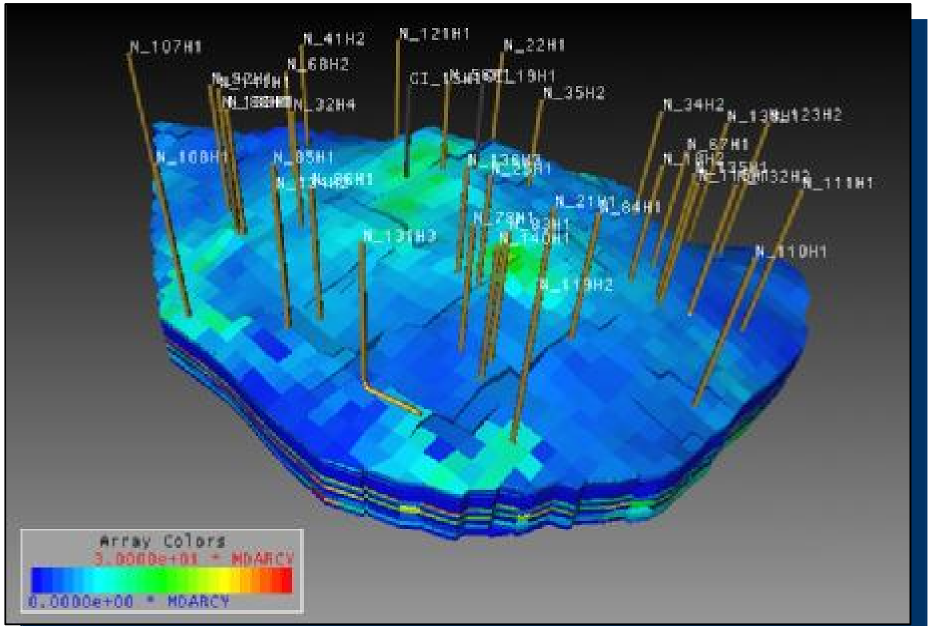
- hydrostatic pressure
- capillary pressure
- pressure
- gradient
- viscosity
- permeability (rock property)
- 'rel perm' (depends on S)
- phase flow

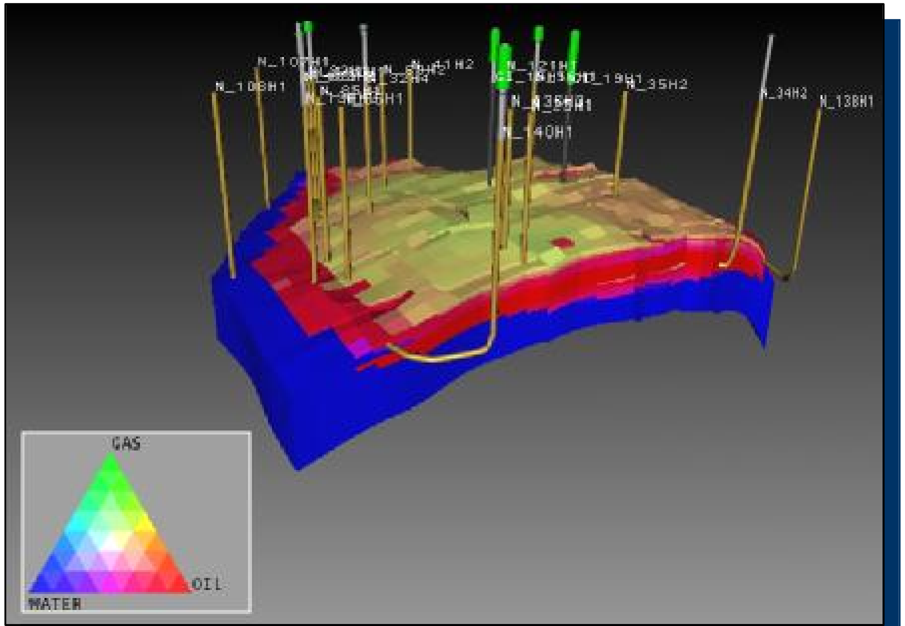
Finite volume discretization

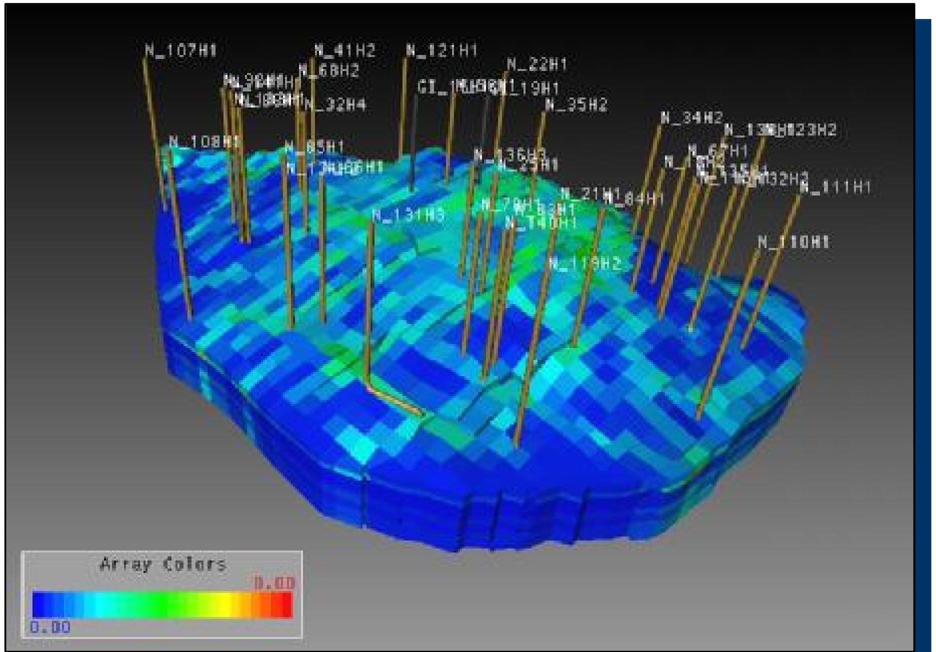
- Mass conservation is important
  - Flux from (two or more point) pressure gradient
  - Mass balance & Volume balance imposed for finite grid block volumes
- One-phase incompressible flow (simplified liquid)
  - Laplace equation for pressure (*elliptic*)
- One phase, compressible flow (gas) (*parabolic*)
- multi-phase, incompressible, flow:
  - Convection equation (*hyperbolic*)

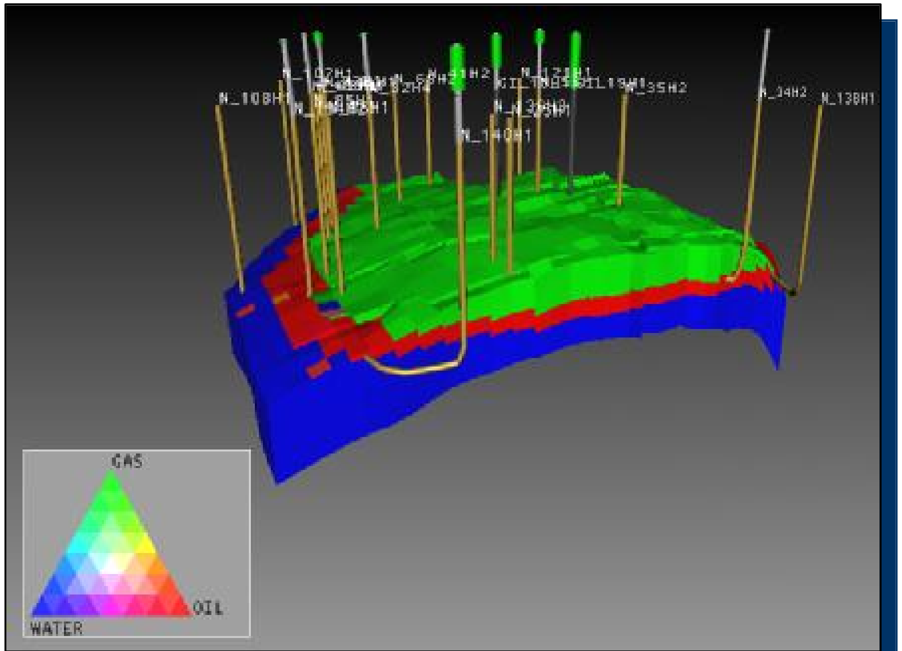
Solve the equations fast and accurately

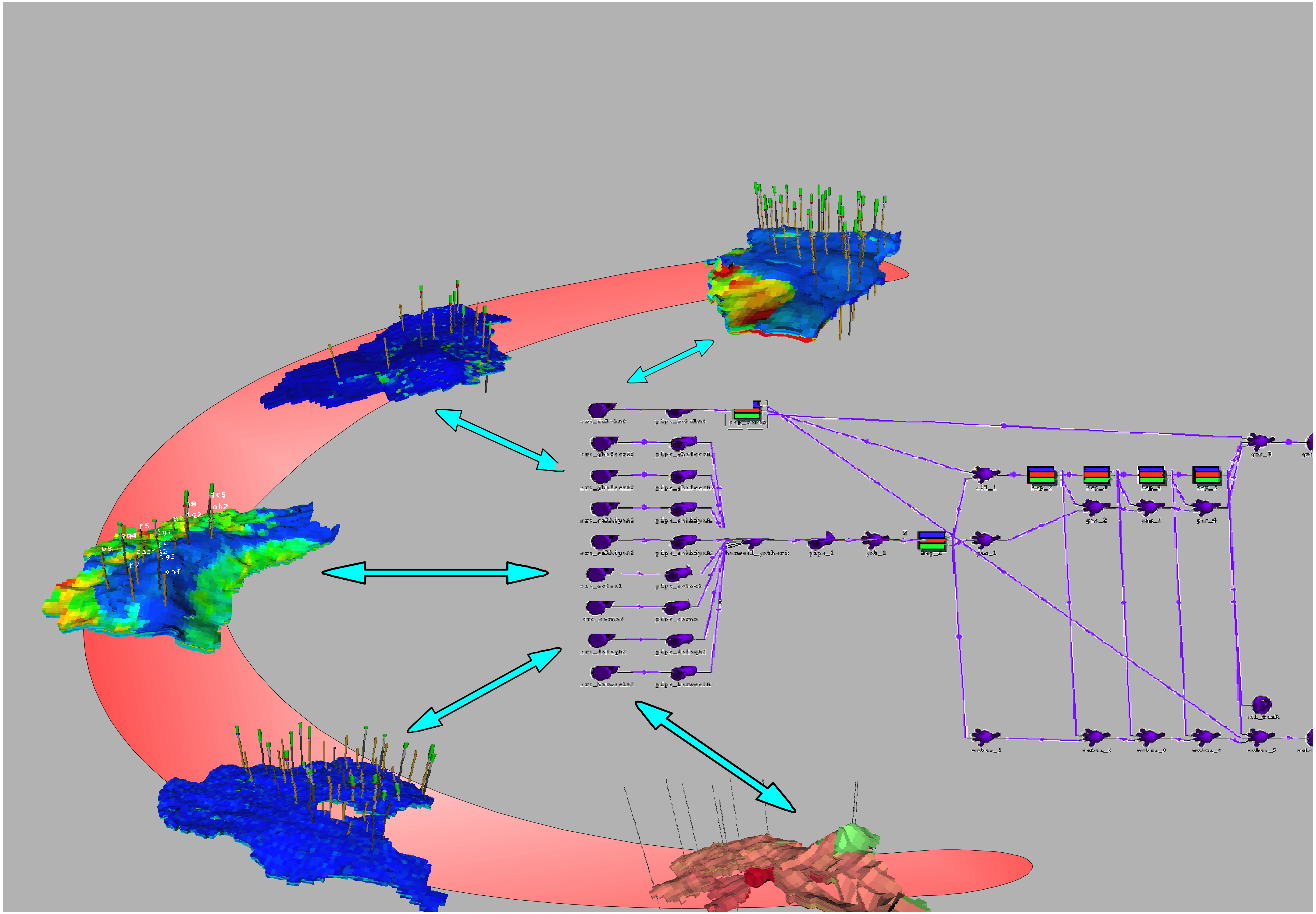
- Newton-Raphson method
- Multi-grid methods, Domain decomposition
- Parallelization


N_107H1
N_41H2
N_121H1
N_22H1
N_68H2
N_32H4
N_35H2
N_34H2
N_108H1
N_85H1
N_21H1
N_84H1
N_131H3
N_140H1
N_110H1
N_119H2
N_111H1
Array Colors
3.0000e+01 * MDARCY
0.0000e+00 * MDARCY


N_108H1
N_35H2
N_140H1
N_34H2
N_138H1
GAS
OIL
WATER


N_107H1
N_41H2
N_121H1
N_22H1
N_68H2
N_32H4
N_35H2
N_34H2
N_108H1
N_85H1
N_67H1
N_131H3
N_140H1
N_84H1
N_110H1
N_119H2
N_111H1
Array Colors
0.00
0.00


N_108H1
N_35H2
N_140H1
N_34H2
N_138H1
GAS
OIL
WATER



- Exploration
- Production
- **Production Operations**

The real production of oil – and gas

A Stopping Time Problem: Production Well Testing

The Mathematics of Engineering

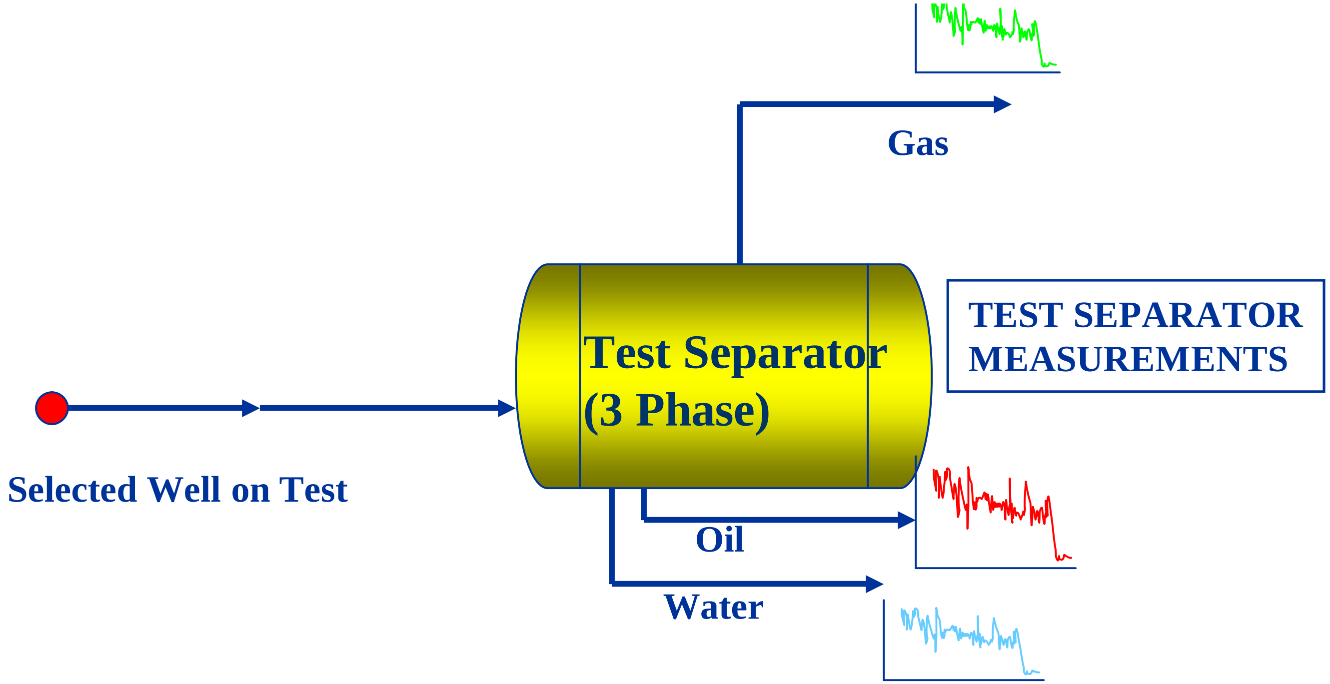
Offshore ‘Champion’ Field in Brunei

transportation tubing

well heads

headers

separators



well testing traditionally: physical model about separation process in separator.
Test times: >= 48 hours, many 'rejections'

Well testing solved as stopping time problem using only separator outputs:

Test times: <= 3 hours; hardly any rejections

Let $\alpha, \beta, \gamma, T_{minimal} \in \mathbb{R}^{+}$ and $k_1, k_2 \in \mathbb{N}$, $k_1 < k_2$

$$T_{candidate} = \{k \in \mathbb{N} \mid \beta \geq max_{l \in \{k_1, k_2\}} |\mathcal{A}(Q)(k) - \mathcal{S}^{l}(\mathcal{A}(Q))(k)|\}$$

backward shift operator

production

well type specific Algorithm

$$T_{candidate} = \bigcup_{i=1}^{N_{candidate}} T^{i}_{candidate}$$

For some $i \in \{1, N_{candidate}\} \subset \mathbb{N}$,

$$T^{i}_{select} = \{k \in \mathbb{N} \mid |\mathcal{F}(\mathcal{A})(k)| \leq \alpha \ , \ k \in \mathbb{T}^{i}_{candidate}\}$$

$$T^{i}_{select} = \bigcup_{j=1}^{N^{i}_{select}} T^{i,j}_{select}$$

Reorder number the $N_{select} := \sum_{j=1}^{N_{candidate}} N^{j}_{select}$ selected periods using the index function $m : \{(i,j) \mid 1 \leq i \leq N_{candidate} \ , \ 1 \leq j \leq N^{i}_{select}\} \rightarrow \{1, \ldots, N_{select}\}$, defined by

$$m_{ij} = \sum_{l=1}^{i-1} N^{l}_{select} + j \ ; \ i > 1$$

$$m_{ij} = j \ ; \ i = 1$$

The m_{ij}-th selected period is:

$$T_{select}^{m_{ij}} := \{\tau_b^{m_{ij}}, \tau_e^{m_{ij}}\}$$

The total selected period is:

$$T_{select} = \bigcup_{m_{ij}=1}^{N_{select}} T_{select}^{m_{ij}}$$

Define

$$\Omega(k) = \{\tau_b^{m_{i_1 j_1}}, \tau_e^{m_{i_1 j_1}}\} \cup \cdots \cup \{\tau_b^{m_{i_{n-1} j_{n-1}}}, \tau_e^{m_{i_1 j_1}}\} \cup \{\tau_b^{m_{i_n j_n}}, k\} \subseteq T_{select}$$

Then

$$T_{STOP} = min\{k \mid k \in T_{select} \bigwedge \mathcal{P}_{time}(Q)(k) < \gamma \bigwedge |\Omega(k)| \geq T_{minimal}\}$$

time statistic

. Exploration

. Production

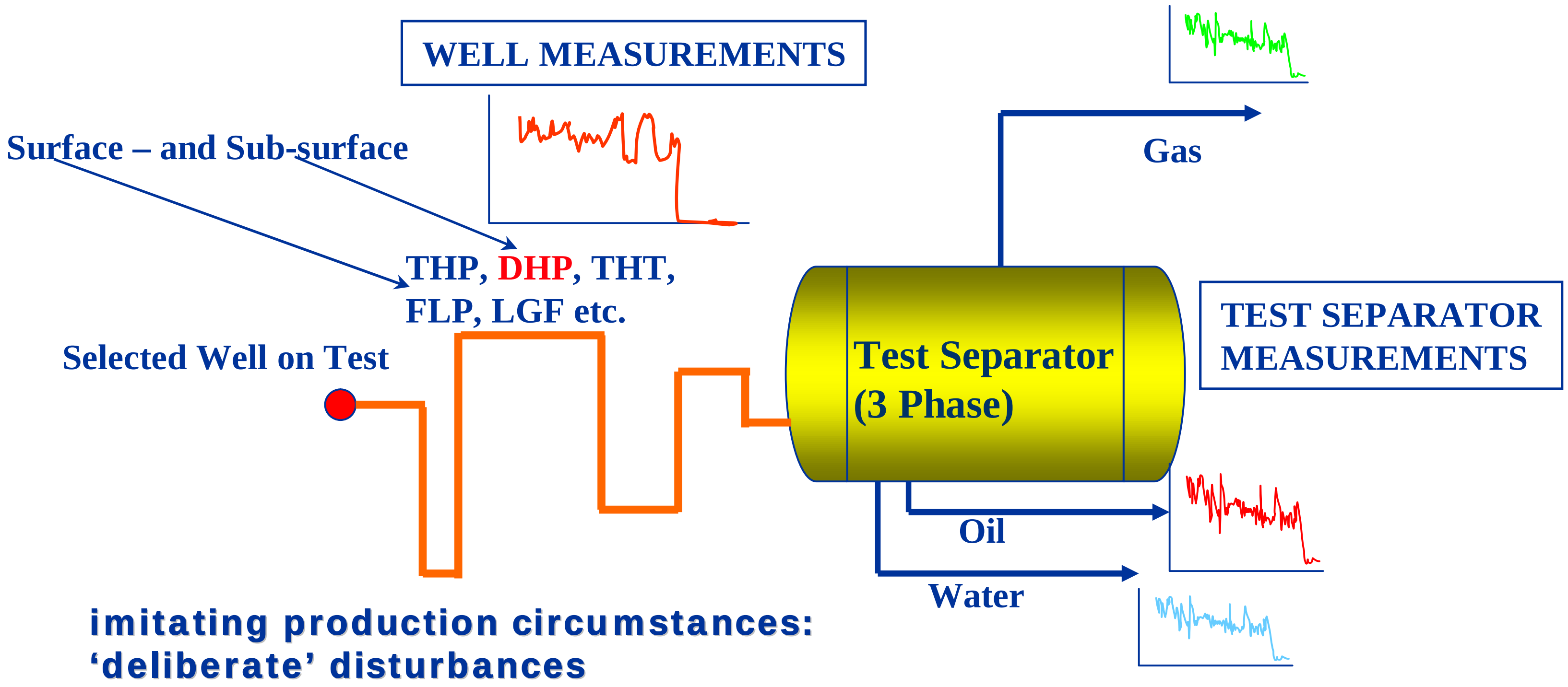
. Production Operations

. **A First Paradigm Shift**

End of Physical Models Monopoly!

Dynamic Systems: Real-Time Production Monitoring

- **everything constructed from the DATA**
- **production system ‘anonymous’ data generator**



Build Models using Well Test Data

$$\psi \in \mathbb{X}$$

$$\psi = \begin{pmatrix} q \\ u \end{pmatrix}$$

well production (q)

input(s) (u)

Assume that $\mathbb{X}$ is a compact metric space.

Identify from well test experiment $\mathbf{F} : \mathbb{X} \to \mathbb{X}$

$$\mathbf{F}(\psi) = \begin{pmatrix} \mathbf{f}(\psi) \\ \mathbf{S}(u) \end{pmatrix}$$

shift map ($\mathbf{S}$)

$\mathbf{F}$ viewed as dynamical system: ξ follows ψ if $\xi = \mathbf{F}^n(\psi)$ for some $n = 1, 2, \ldots$

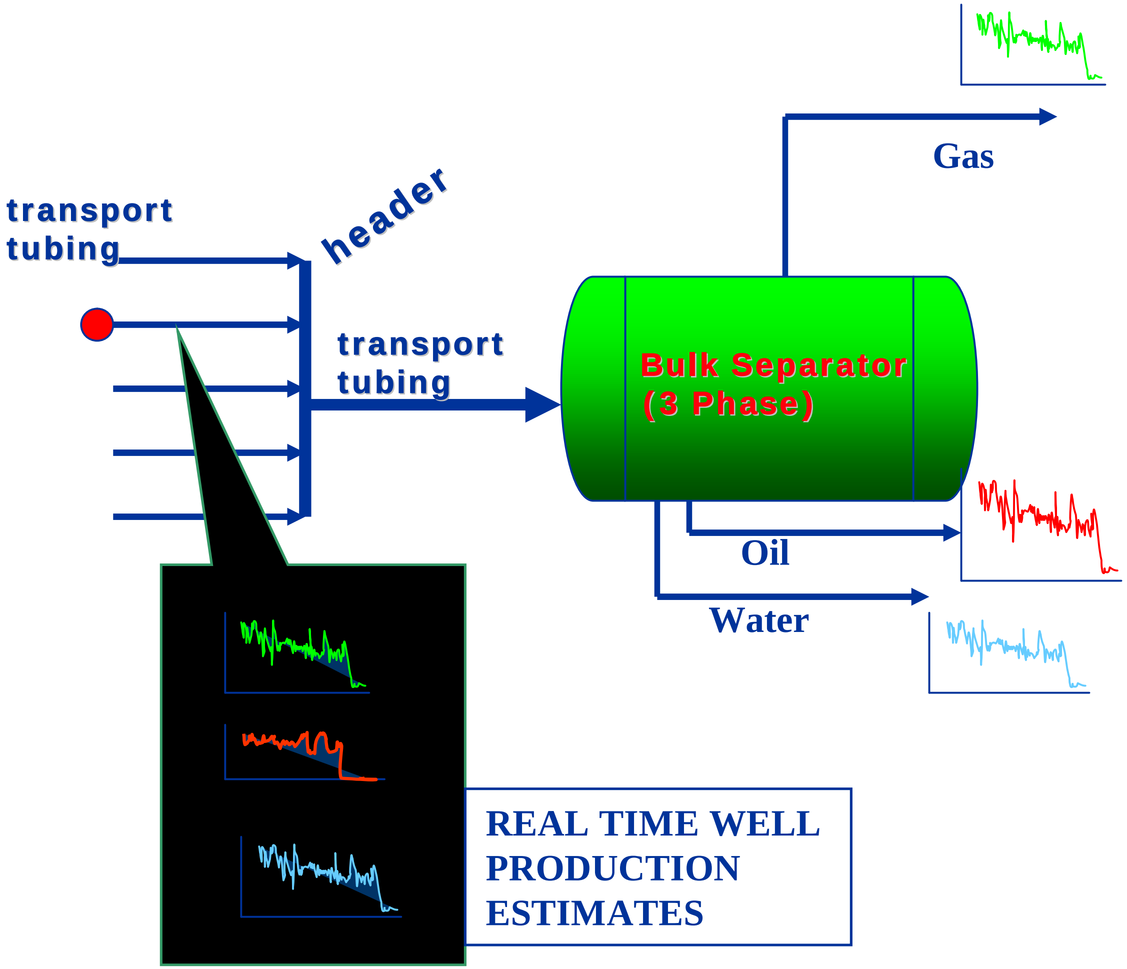
Identify $\mathbf{F}$ and its iterates with their graphs in $\mathbb{X} \times \mathbb{X}$.

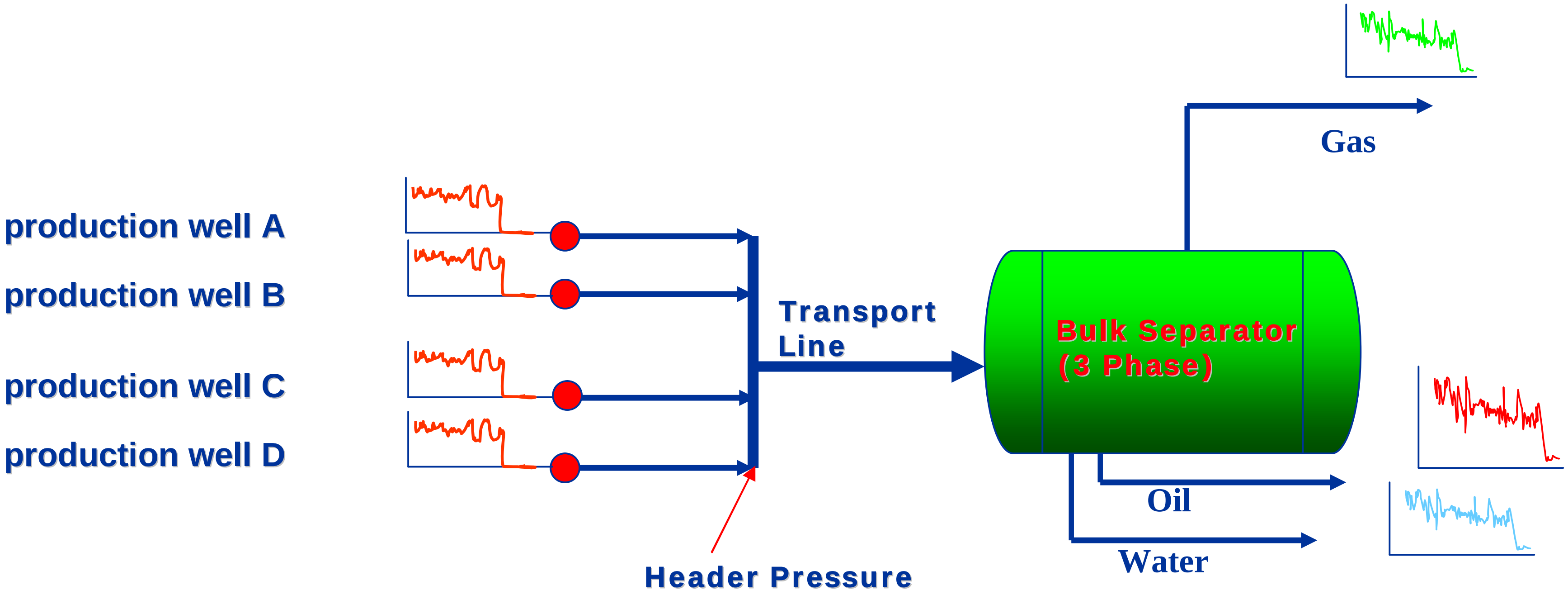
$$\mathcal{O}\mathbf{F} = \bigcup_{n=1}^{\infty} \mathbf{F}^n$$

$$(\xi, \psi) \in \mathcal{O}\mathbf{F} \Leftrightarrow \xi = \mathbf{F}^n(\psi) \text{ for some } n = 1, 2, \ldots$$

$$\mathcal{O}\mathbf{F}(\psi) = \{\mathbf{F}(\psi), \mathbf{F}^2(\psi), \ldots\}$$ **positive orbit**

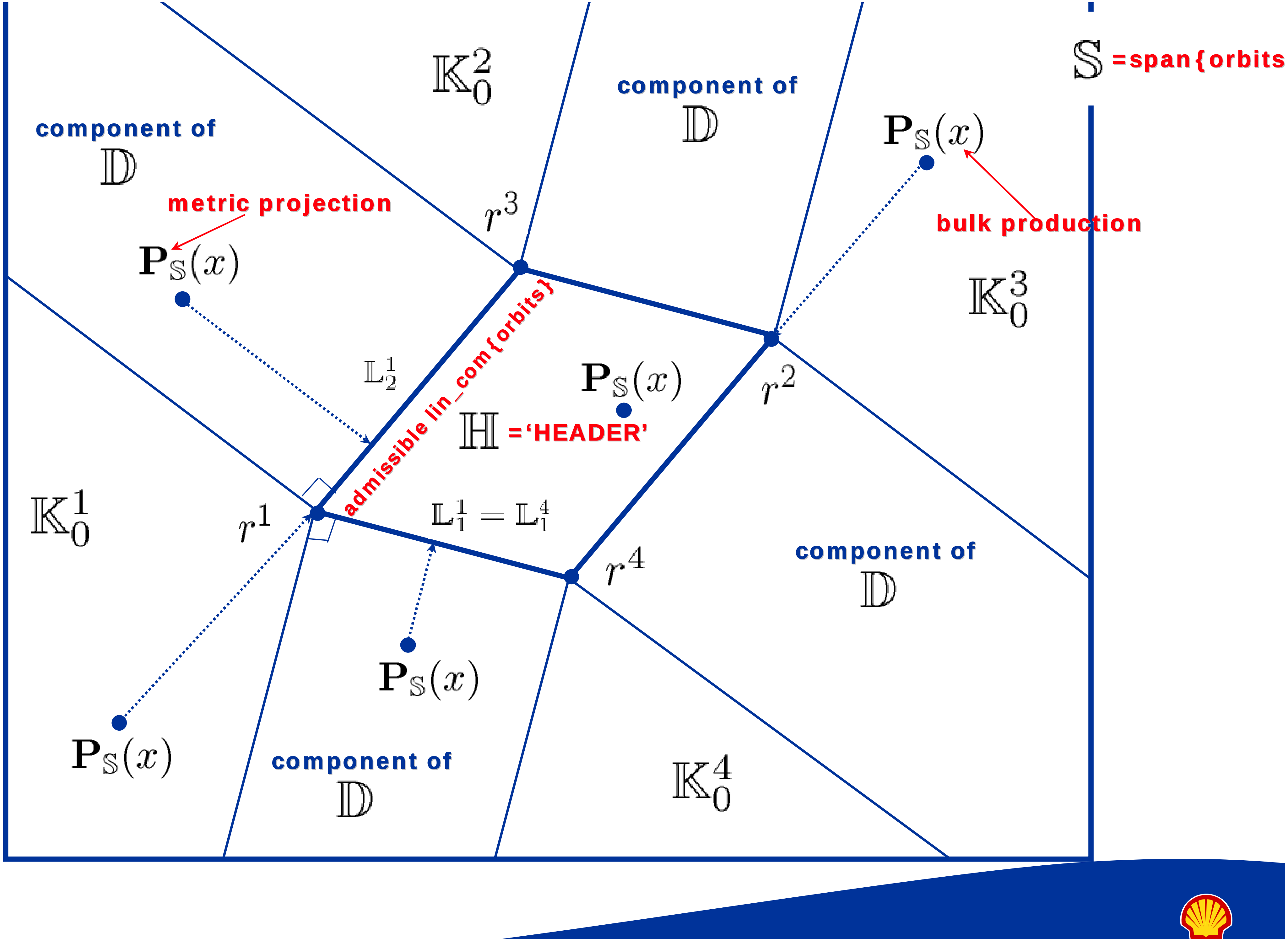
the orbits for each well give the production estimates for each well





Daily Reconciliation

compare and adjust individual Estimates against bulk metering.



$\mathbb{S}$ =span{orbits
component of $\mathbb{D}$
$\mathbb{K}_0^2$
component of $\mathbb{D}$
$\mathbf{P}_{\mathbb{S}}(x)$
metric projection
r^3
bulk production
$\mathbb{K}_0^3$
admissible lin_com{orbits}
$\mathbb{L}_2^1$
r^2
$\mathbb{H}$ =‘HEADER’
$\mathbb{K}_0^1$
r^1
$\mathbb{L}_1^1 = \mathbb{L}_1^4$
r^4
component of $\mathbb{D}$
component of $\mathbb{D}$
$\mathbb{K}_0^4$

$$\mathbb{V}_\epsilon = \{(\psi_1, \psi_2) \in \mathbb{X} \times \mathbb{X} \mid d(\psi_1, \psi_2) \leq \epsilon\}$$

$$\epsilon\text{-chain } \{\psi_n\} : \psi_{n+1} \in \mathbb{V}_\epsilon(\mathbf{F}(\psi_n))$$

$$\mathcal{C}\mathbf{F} = \bigcap \{\mathcal{O}(\mathbb{V}_\epsilon \circ \mathbf{F}) \mid \epsilon > 0\}$$ **(chain recurrent set)**

$$\xi \in \mathcal{C}\mathbf{F}(\psi) \Leftrightarrow \forall \epsilon \ \exists \ \epsilon\text{-chain beginning at } \psi \text{ and ending at } \xi$$

number of wells

bulk production

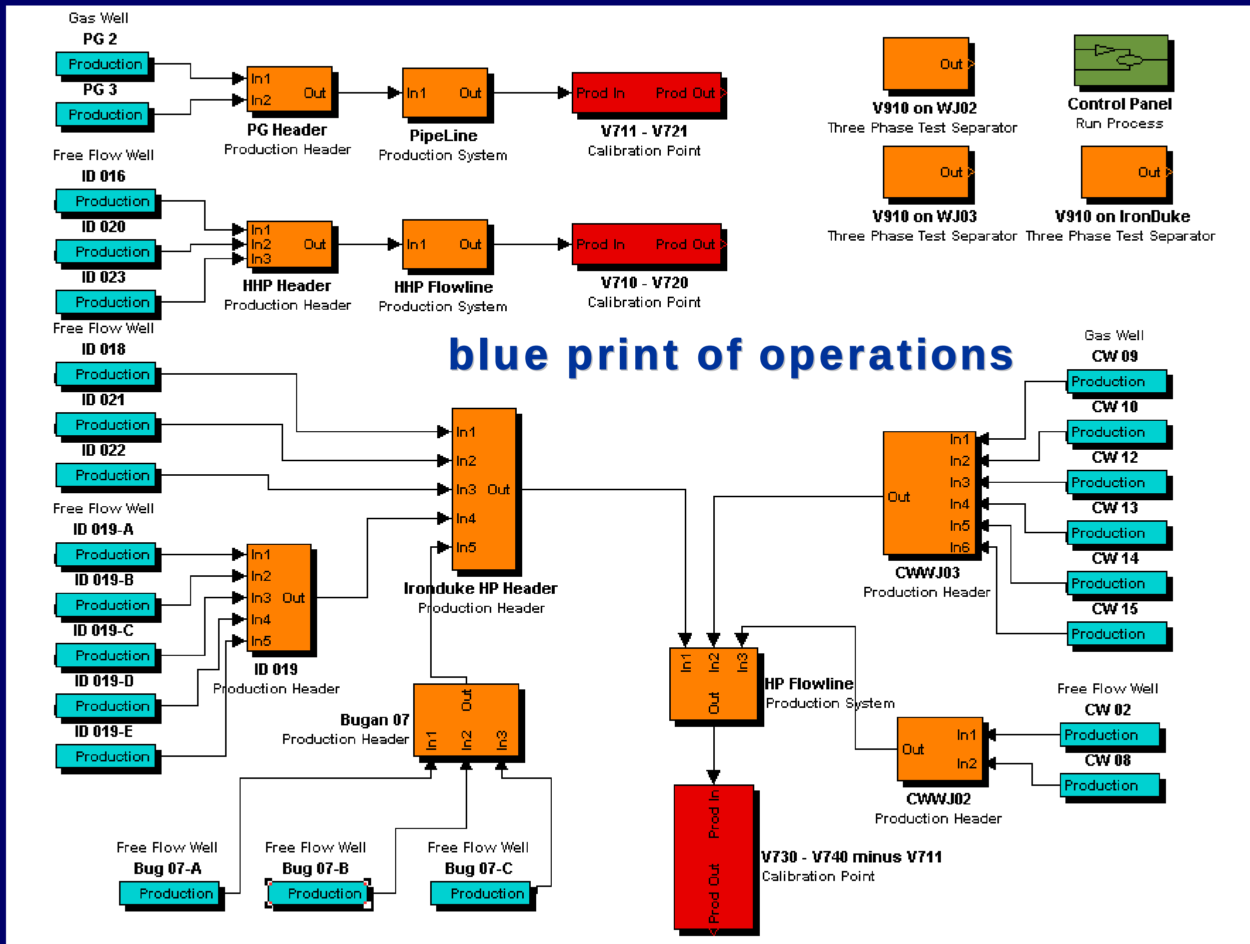
$$q \approx \sum_{j=1}^{N} \rho_j q_j$$

reconciliation coefficients

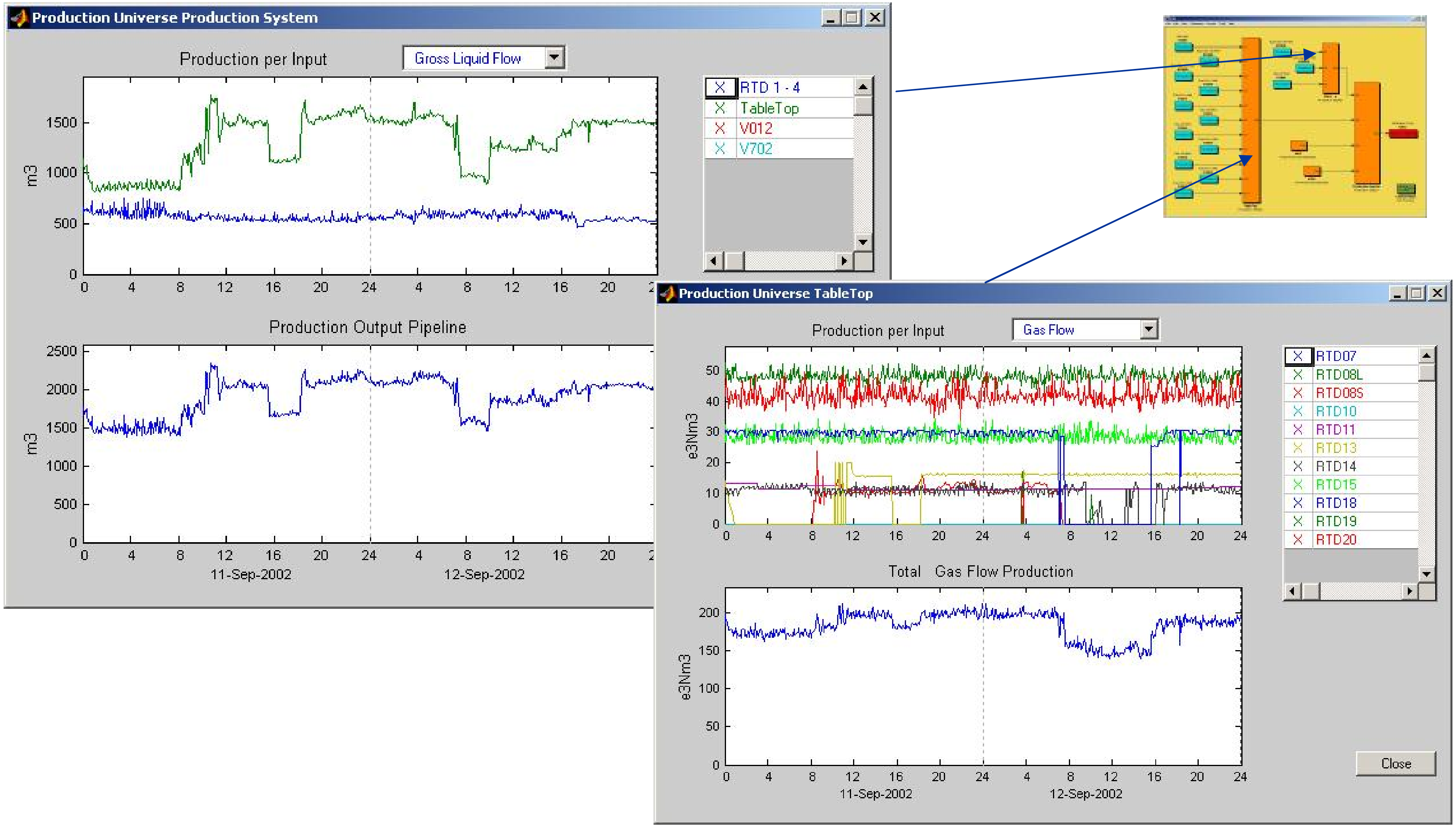
adapt $$\hat{\psi}_j = \begin{pmatrix} \rho_j q_j \\ u \end{pmatrix}$$

consistency $$\mathcal{O}\mathbf{F}_j(\hat{\psi}_j) \subseteq \mathcal{C}\mathbf{F}(\hat{\psi}_j)$$ **otherwise re-test**

Brunei – Iron Duke and Champion Oil Platforms



blue print

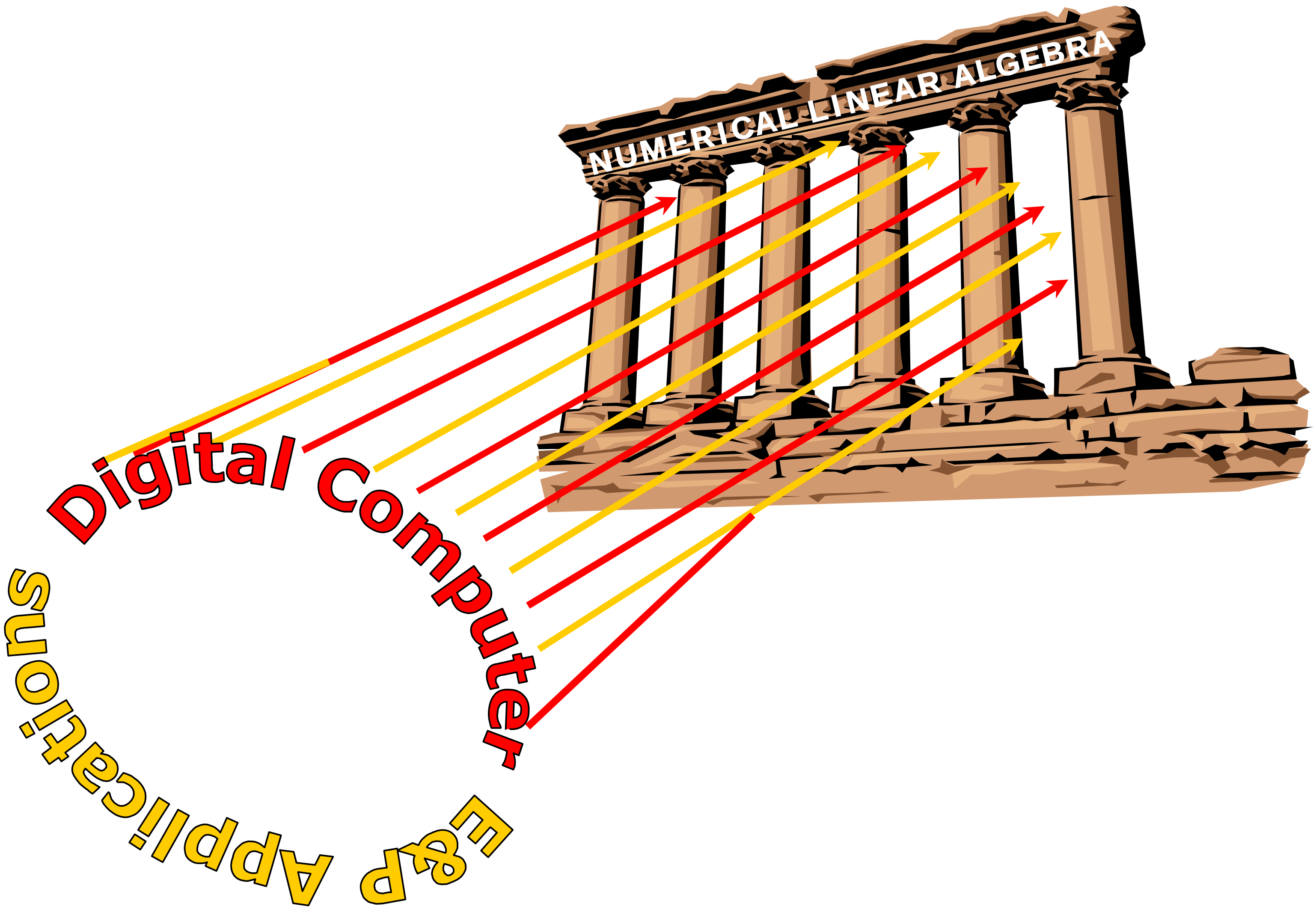


Production Universe Production System
Production per Input
Gross Liquid Flow
RTD 1 - 4
TableTop
V012
V702
m3
Production Output Pipeline
11-Sep-2002
12-Sep-2002
Production Universe TableTop
Production per Input
Gas Flow
e3Nm3
RTD07
RTD08L
RTD08S
RTD10
RTD11
RTD13
RTD14
RTD15
RTD18
RTD19
RTD20
Total Gas Flow Production
Close

- Exploration
- Production
- Production Operations
- A First Paradigm Shift
- **A Second Paradigm Shift**

End of Digital Computer Monopoly?

algebraic structures

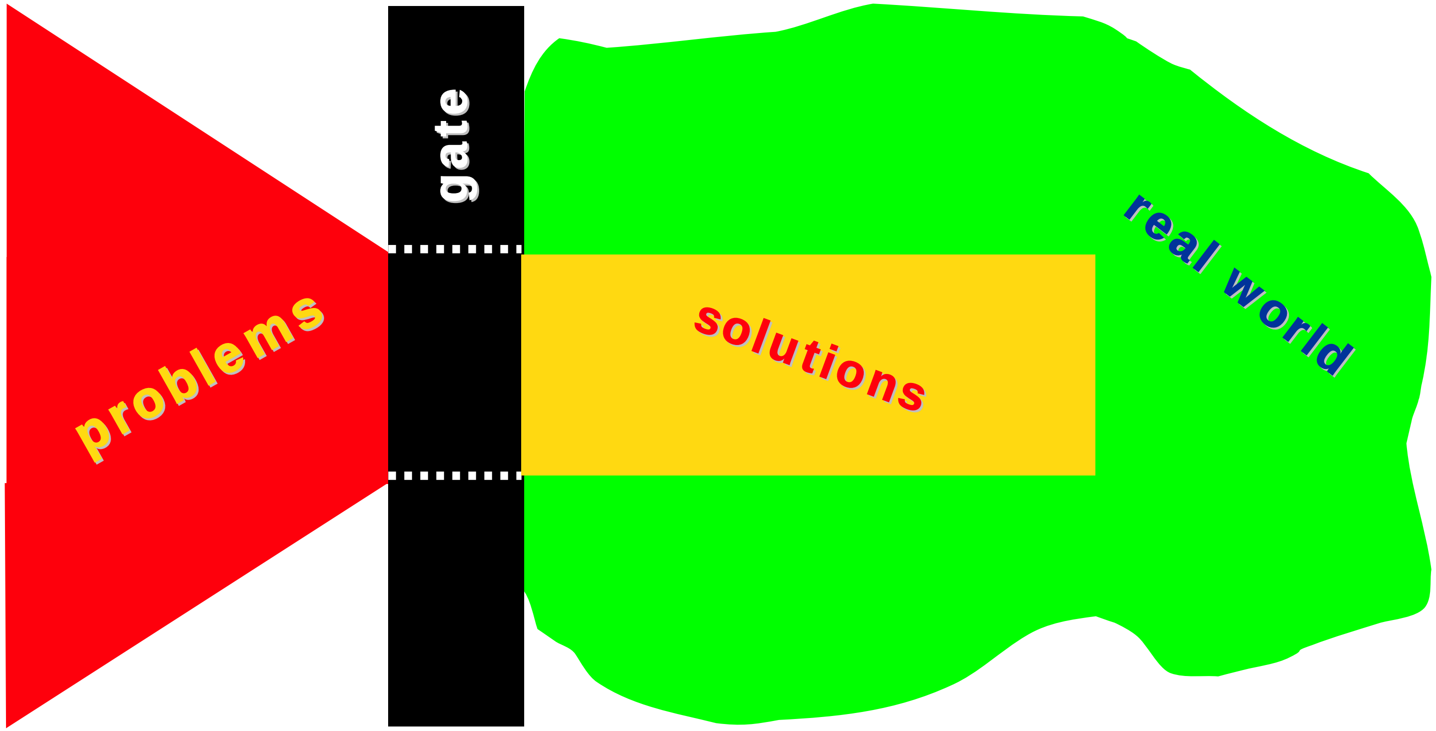


NUMERICAL LINEAR ALGEBRA
Digital Computer
E&P Applications

Observation

*Despite their original variability, virtually all 'digital computer' applications sooner or later have to fit into the algebraic structure of a **Vector Space** on which **'scalars'** operate that come from the **Field** of real – or complex numbers*

Top View



Is the 'Gate' for at least some Problems too narrow?

Widening the 'Gate':

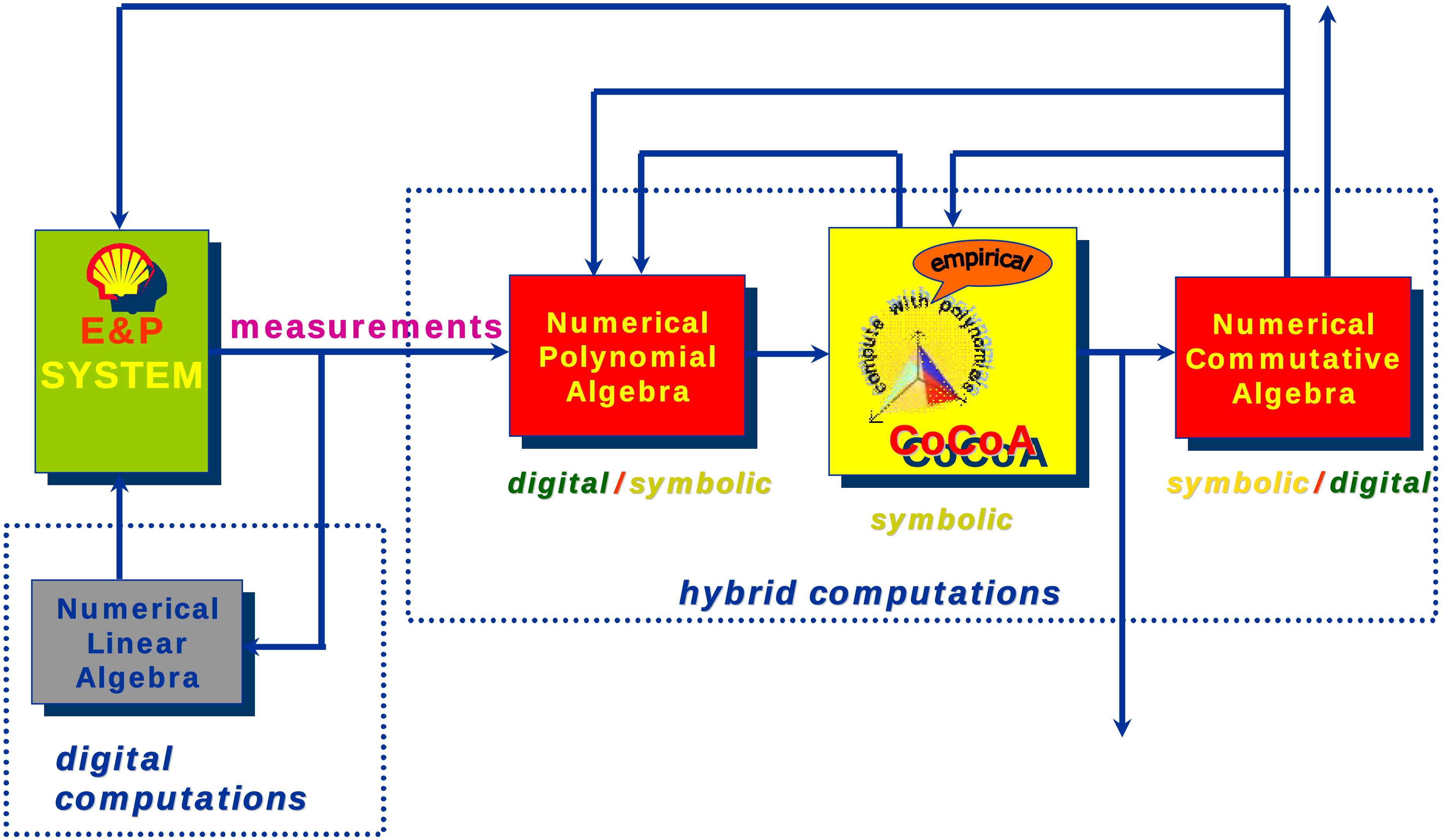
Let the applications to end up in the algebraic structure

of a Module, where the 'scalars' are allowed to come

From an – arbitrary - Ring

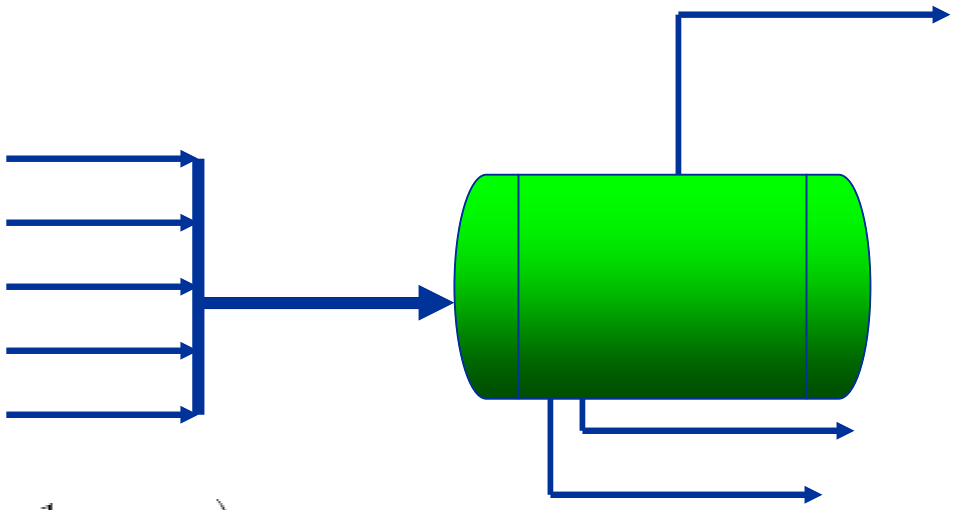
Can this idea be realized?

Computer algebra


E&P
SYSTEM
measurements
Numerical
Polynomial
Algebra
empirical
compute with polynomials
CoCoA
Numerical
Commutative
Algebra
digital / symbolic
symbolic
symbolic / digital
hybrid computations
Numerical
Linear
Algebra
digital
computations

$x_i \ (i = 1, \ldots, m) \ m \geq n$

inputs
(well measurements)



y **total production**

$y_i \ (i = 1, \ldots, n)$

well productions

Consider the Polynomial Ring $\mathbf{R} = \mathbb{R}[x_1, \ldots, x_m]$

Construct the empirical productions $y, y_1, \ldots, y_n \in \mathbb{R}[x_1, \ldots, x_m]$ **from the Data**

By considering the tuple $W = (y_1, \ldots, y_n, y)$ of the $\mathbf{R}$-module $\mathcal{M}$

Find the DECOMPOSITION of y given by:

$$y = h_1 y_1 + \cdots + h_n y_n$$

where

$$h_i \in \mathbb{R}[x_1, \ldots, x_n]$$

- no unique mathematical solution
  - Syzygy module $Syz_{\mathbf{R}}(y_1, \ldots, y_n)$ gives information how many representations in $y_1, \ldots, y_n$ exist
  - only one of them is physically realizable
    - changing term ordering on basis of physical significance of the x_i

      e.g. "all surface quantities precede all sub-surface ones"
- h_i in representation of y reveal
  - interrelationships among the wells

    $h_i(x_k, \cdots)$, j well number $i \neq j$
  - surface – sub-surface relationship=> optimization

    h_i depending on sub-surface inputs=>'ultimate recovery'

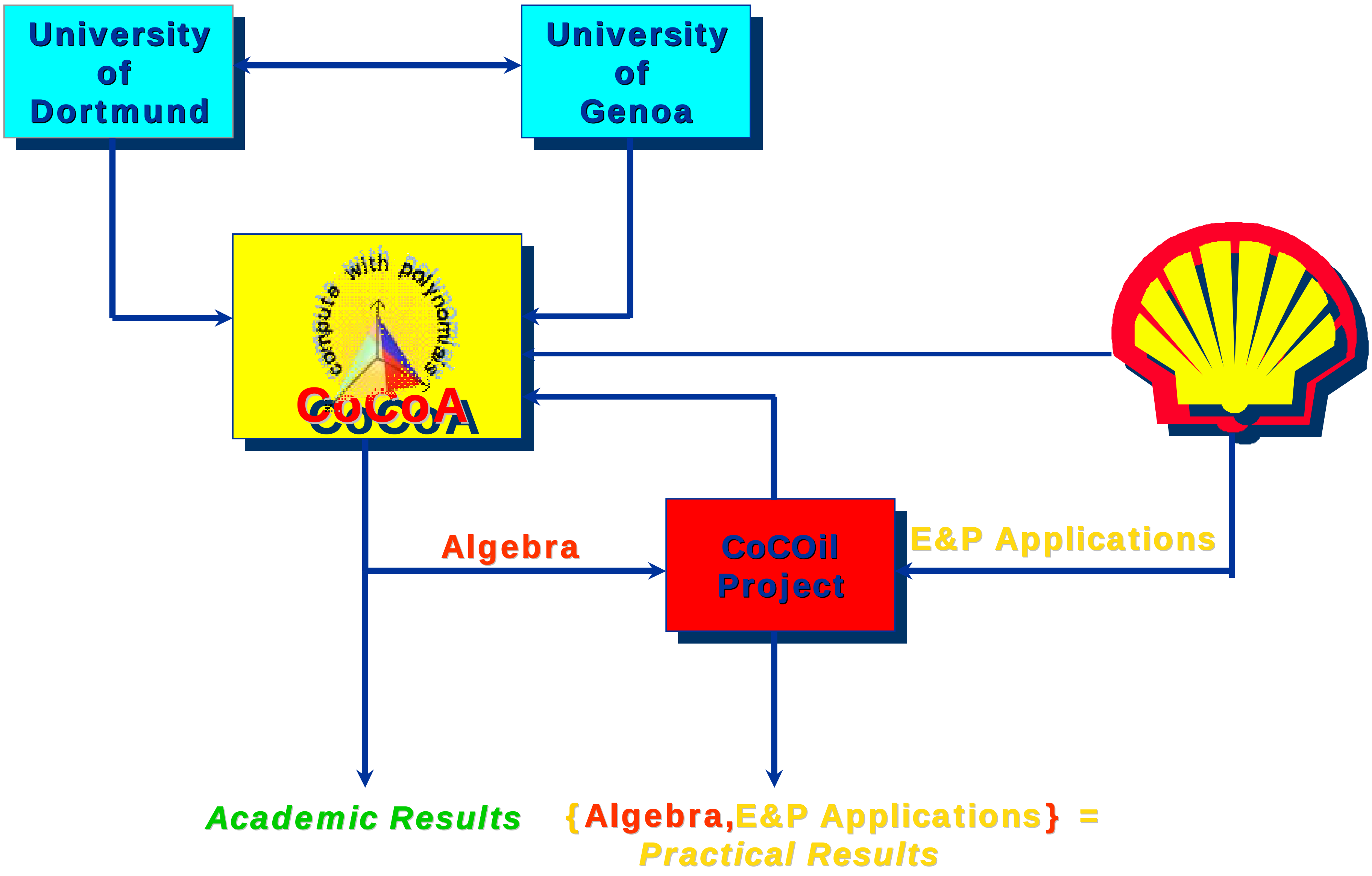
Some Considerations

- ‘hardware’ elements of production system have context dependent mathematical equivalents
    - Header as Convex Set, and as Ideal
- abstract description of ‘practice’
    - term order to capture work sequence of ‘production operators’
- ‘user’ does NOT impose complexity on the system through the choice of a – physical – model
- system reveals its own complexity through the measurements
- ‘scale’ of measurements and that of physical model may not be compatible
- -dynamic- models extracted from the measurements always on the ‘right’ scale

- Exploration
- Production
- Production Operations
- A First Paradigm Shift
- A Second Paradigm Shift
- Algebraic Encounters of the First Kind
- **The CoCOil Research Programme**

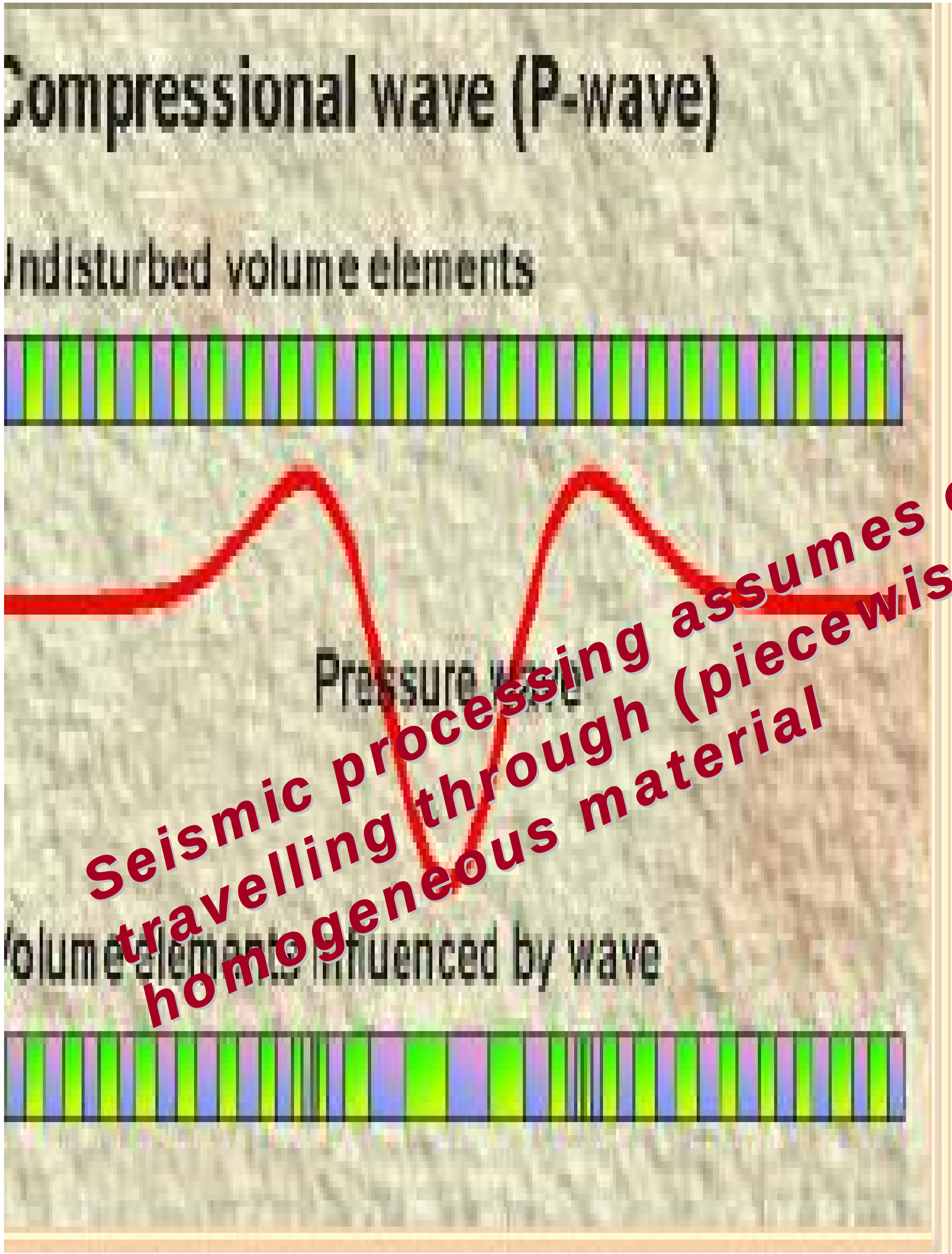
  A sequence of {Algebra, E&P Application} pairs

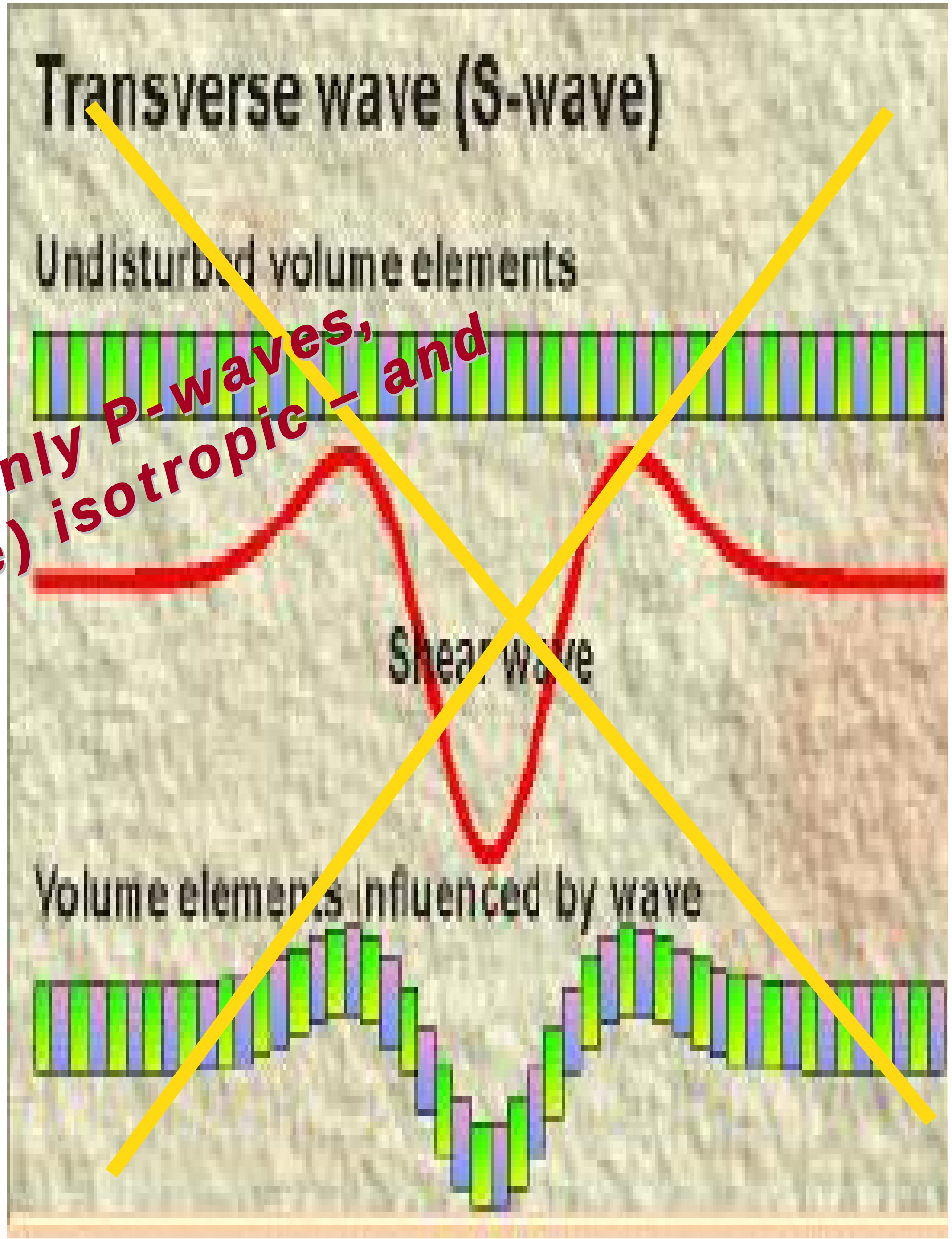
We proudly present….


University of Dortmund
University of Genoa
CoCoA
Algebra
CoCOil Project
E&P Applications
Academic Results
{Algebra,E&P Applications} =
Practical Results

Algebraic Subject	E & P Application
Syzygies	**Interrelationships** Sub-surface ó Surface Relationship: Ultimate Recovery
Differential Gröbner Basis	**Dynamical Systems** Including long-term changes=>Forecasting, Reservoir management. Special activity: Good Slugs, using the energy generated by slugs for production – and exploration (see last pair) applications
Elimination Theory	**www2.m** Acronym for 'Where, when, what to measure'. Minimal requirements technical infra structure.
Invariant Theory	**Generic elements** Global exchange of information
Homotopy	**Test versus Production** The change from the test - to the production situation for a well is viewed as a continuous deformation of the well test model
Automated Theorem Proving	**Diagnostics and Decisions** Including relationships between processes that run on different time scales, e.g. early recognition of building-up water break through. Subject may be considered as next generation Artificial Intelligence.
Computational Homology	**Surface characterization** Surface characterization of sub-surface through computation of homology groups. Of particular importance for last pair.
D - Modules	**Non-seismic Exploration** This application is possible since this algebraic subject allows the consideration of spatial variation. This pair is coupled with the first – and second pair.

Backup slides





Seismic processing assumes only P-waves, travelling through (piecewise) isotropic _ and homogeneous material

Deviated, branched wells (multi-zone wells)

Difficult to align grid lines with a non-vertical well paths

Locally refined grids, for high resolution near the wells