Algebraic Oil

Martin Kreuzer

Fakultät für Informatik und Mathematik

Universität Passau

martin.kreuzer@uni-passau.de

Universitetet i Bergen

(Norway)

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Hennie Poulisse

(Shell Int. Exploration & Production)

`Hennie.Poulisse@shell.com`

Your theory is crazy,
but it's not crazy enough to be true.
(Niels Bohr)

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1 – Modeling Oil Production

You've got to be very careful
if you don't know where you are going,
because you might not get there.
(Yogi Berra)

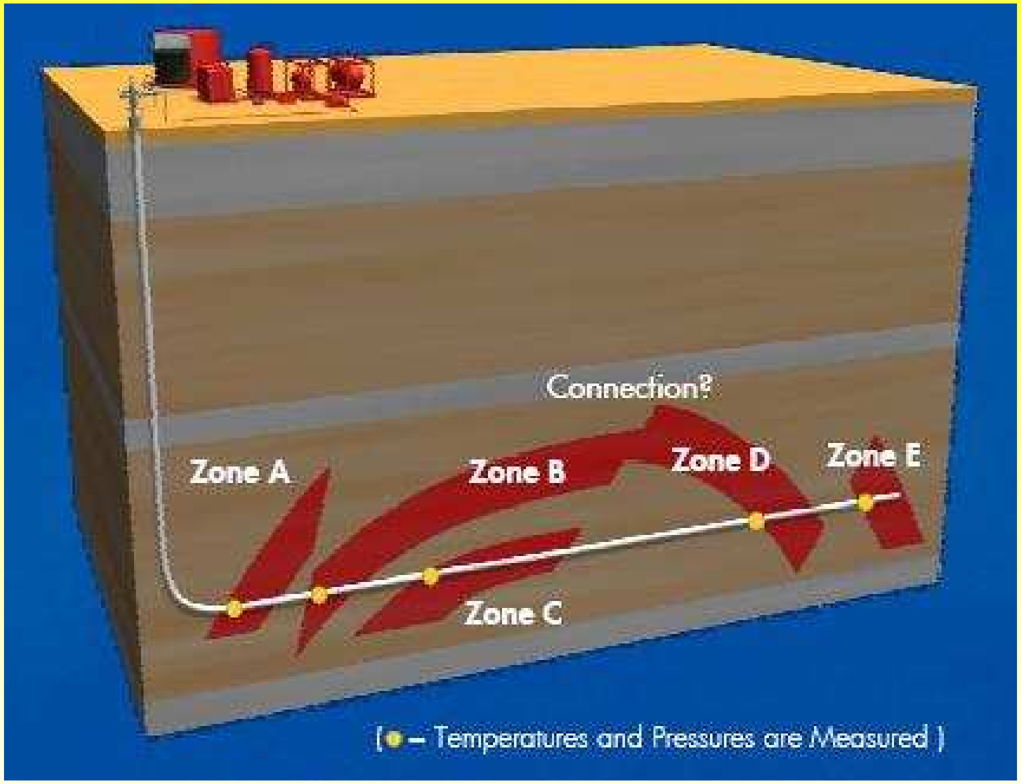
The **Algebraic Oil Research Project** is a collaboration between the **Department of Exploratory Research** at **Shell International Exploration and Production (SIEP)** and the chair of **Symbolic Computation** at **Passau University**.

The basic idea is to introduce methods of **symbolic computation** (computer algebra) to solve some problems in oil production and exploration.

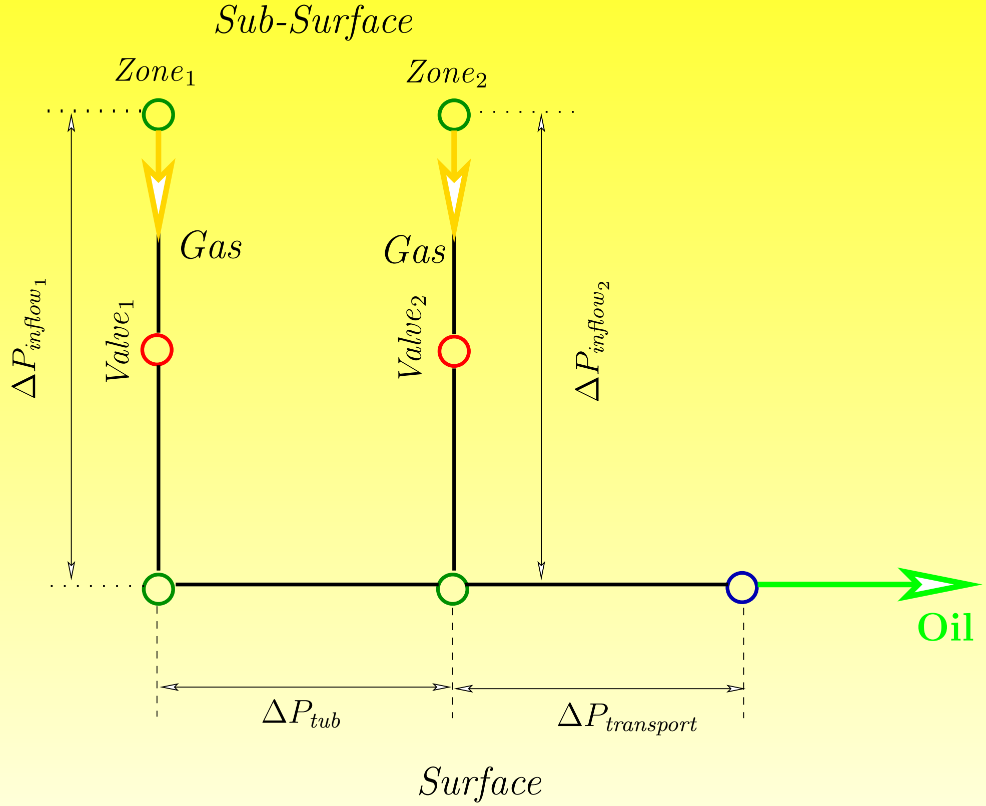
These new techniques are part of an emerging area called **Approximate Commutative Algebra** or **hybrid computation** or **symbolic-numeric computation**.

Problem 1: Find simple **polynomial models** which describe the behaviour of an oil production system over extended periods of time.

Structure of an Oil Well



Schematic Representation of an Oil Well



Basic Tenet 1: Use only the available data!

$$\begin{aligned} x_1 &: \Delta P_{inflow_1} \\ x_2 &: \Delta P_{inflow_2} \\ x_3 &: Gas\ production \\ x_4 &: \Delta P_{tub} \\ x_5 &: \Delta P_{transport} \end{aligned}$$

Basic Tenet 2: Assume that no a priori model is available to describe the production of the well in terms of measurable physical quantities.

Problem 2: Find an algebraic model of the production in terms of these quantities which specifically models the interactions occurring in this production unit.

Why Polynomials?

(1) Many important physical laws are given by polynomials.

(2) In order to achieve a high **predictive power** of a model one has to use a **rigid** set of functions.

(3) But at the same time we prefer to impose **no model structure**, i.e. we do not want to make detailed assumptions about the support of the model polynomials.

(4) Many functions can be approximated well by polynomials.

(5) The coefficients of **simple** model polynomials have physical interpretations. We can get **quantitative predictions**!

2 – Approximative Commutative Algebra

All those who believe in telekinesis,
raise my hand.
(Steven Wright)

$P = \mathbb{R}[x_1, \dots, x_n]$ polynomial ring over the field of real numbers $\mathbb{R}$

$\mathbb{X} = \{p_1, \dots, p_s\} \subset \mathbb{R}^n$ finite set of (empirical) points

Definition 2.1 Given a **threshold number** $\varepsilon > 0$, we say that a polynomial $f \in P$ **vanishes ε-approximately** at a point $p \in \mathbb{R}^n$ if $\|f(p)\| < \varepsilon$.

Problem: Every polynomial with small coefficients vanishes ε-approximately at p!

Definition 2.2 **(a)** For every $f \in P$, we let $\|f\|$ be the (euclidean) **norm** of the coefficient vector of f.

(b) A polynomial is called **unitary** if $\|f\| = 1$.

Thus we are looking for unitary polynomials which vanish ε-approximately at the points of $\mathbb{X}$.

Problem: If one adds such a polynomial to a model, one obtains again a model.

Definition 2.3 **(a)** A polynomial of the form $x_1^{\alpha_1} \cdots x_n^{\alpha_n}$ with $\alpha_i \geq 0$ is called a **term.**

(b) A finite set of terms $\mathcal{O}$ is called an **order ideal** if $t \in \mathcal{O}$ and $t' \mid t$ implies $t' \in \mathcal{O}$.

(c) The set $\partial\mathcal{O} = (x_1\mathcal{O} \cup \cdots \cup x_n\mathcal{O}) \setminus \mathcal{O}$ is called the **border** of $\mathcal{O}$.

Plan: **(a)** Use an interpolation space whose basis consists of the terms of an order ideal.

(b) The interpolation space should be a residue class ring of P modulo an **approximate vanishing ideal**, i.e. an ideal generated by unitary polynomials vanishing approxiamtely on $\mathbb{X}$.

Definition 2.4 Let $\mathcal{O} = \{t_1, \ldots, t_\mu\}$ be an order ideal and $\partial\mathcal{O} = \{b_1, \ldots, b_\nu\}$ its border.

(a) A set of polynomials $\{g_1, \ldots, g_\nu\}$ of the form $g_j = b_j - \sum_{i=1}^{\mu} c_{ij} t_i$ with $c_{ij} \in K$ and $t_i \in \mathcal{O}$ is called an **$\mathcal{O}$-border prebasis**.

(b) An $\mathcal{O}$-border prebasis is called an **$\mathcal{O}$-border basis** if the residue classes of the terms in $\mathcal{O}$ are a K-vector space basis of $P/\langle g_1, \ldots, g_\nu \rangle$.

The AVI Algorithm

There is an algorithm which takes as input a set of (empirical) points $\mathbb{X} \subset [-1,1]^n$ and computes an order ideal $\mathcal{O}$ and an **approximate $\mathcal{O}$-border basis** $G = \{g_1, \ldots, g_\nu\}$ which vanishes ε-approximately at $\mathbb{X}$.

Here G is called an approximate $\mathcal{O}$-border basis if it is an $\mathcal{O}$-border prebasis and if the coefficients c_{ij} almost satify the equations required for G to be a border basis.

Basic Tenet 3: Do not linearize unnecessarily!
Use the power of polynomial algebra, not just linear algebra!

Why Ideals?

(1) Almost vanishing polynomials have **naturally** the structure of a polynomial ideal: The sum of two almost vanishing unitary polynomials is almost vanishing. If you multiply an almost vanishing polynomial by an arbitrary unitary polynomial, the result is almost vanishing.

(2) The residue classes modulo a polynomial ideal have naturally the structure of a **ring**, i.e. you can add and multiply residue classes.

(3) Starting from a polynomial ideal one can compute higher (more subtle) algebraic invariants which contain deeper structural insights into the physical system.

Does It Work?

Example 2.5 Let us take 2052 data points in $\mathbb{R}^6$ relating to the driving forces of **Zone A** of a certain two-zone well, labelled as above but with an additional indeterminate for inflow valve A.

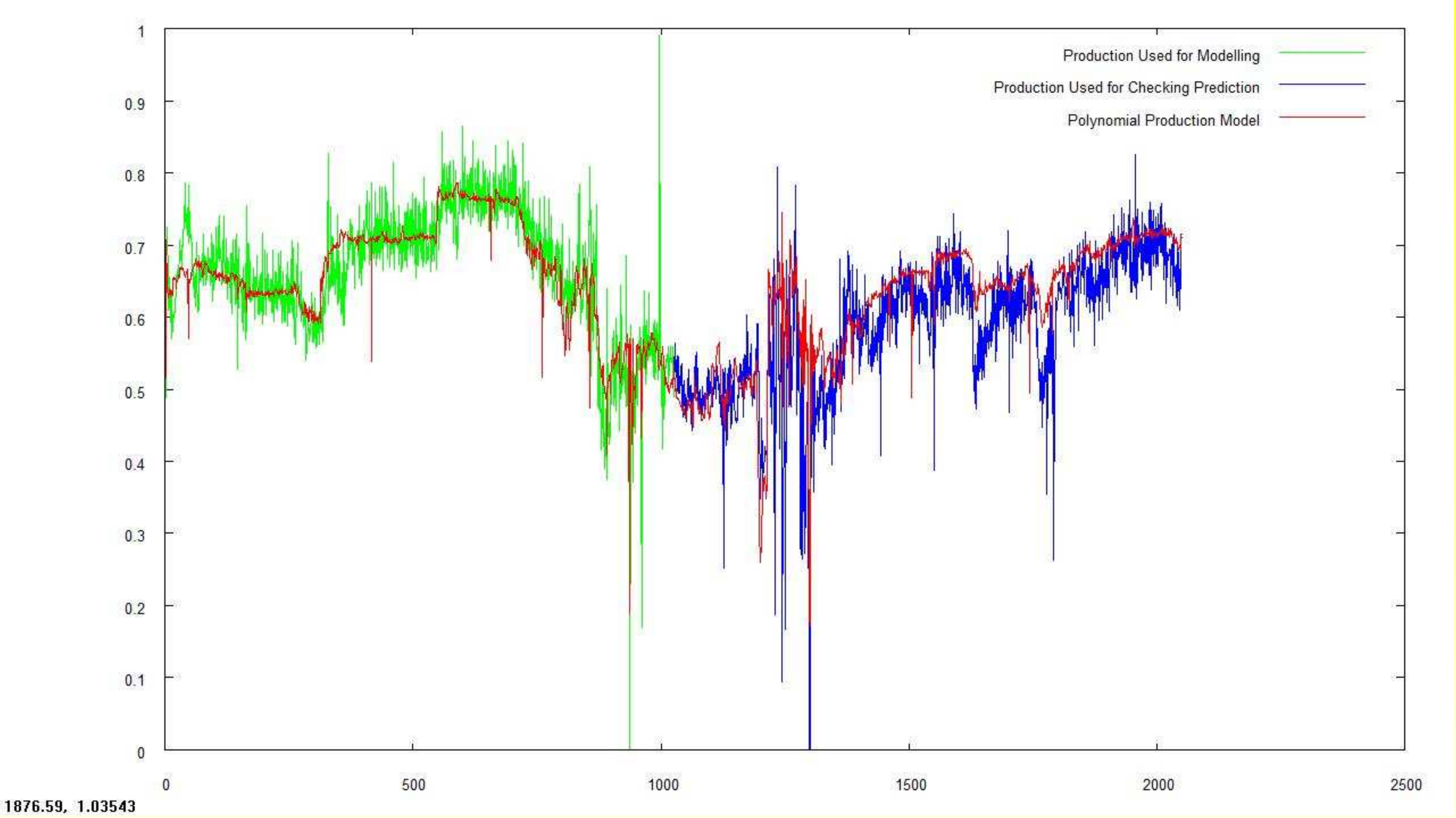
(a) First we use 1026 of the points and the AVI algorithm with $\varepsilon = 0.2$ to get the order ideal $\mathcal{O} = \{1, x_6, x_5, x_4, x_3, x_1, x_6^2, x_1x_6, x_5^2\}$ and an approximate $\mathcal{O}$-border basis.

Then we use a linear projection to write the total fluid production approximately as a linear combination in $\langle \mathcal{O} \rangle_K$ and get

$$-1.69x_5^2 - 0.26\, x_1x_6 - 1.01\, x_3 - 0.11\, x_4 + 1.86\, x_5 - 0.38\, x_6 + 0.58$$

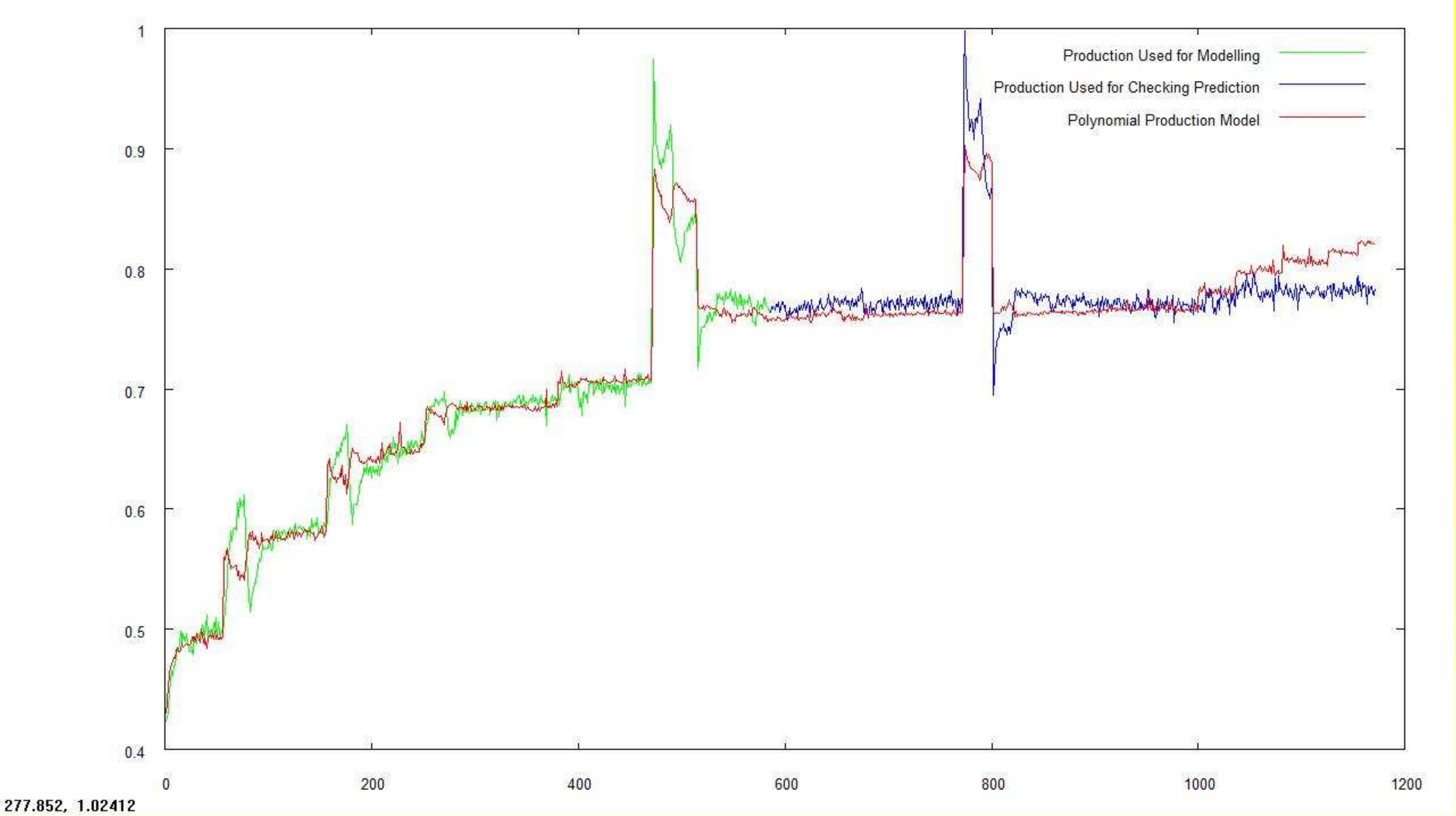
The average evaluation of the polynomial at the points used for the fitting experiment is **0.07**.

Production Modeling Example 1



(b) Similarly, we model 1171 data points for **Zone B** when it is **on test** (i.e. producing alone) and get $\mathcal{O} = \{1, x_6, x_5, x_4, x_1, x_6^2\}$ with production polynomial
$0.04\,x_6^2 + 0.13\,x_1 + 0.62\,x_4 + 0.46\,x_5 + 0.13\,x_6 - 0.23$ and average evaluation error **0.06**.

Production Modeling Example 2



3 – Production Allocation

I really didn't say everything I said.
Half the lies they tell about me aren't true.
(Yogi Berra)

Problem: Several zones of a well are producing together **(commingled production)**. Some of them are influencing each other. Typical interactions are:

(1) The oil produced by one zone may push back the oil trying to flow in at another zone.

(2) The gas which is produced simultaneously with the oil may have stimulating or inhibiting effects **(liquid-gas relationship)**.

(3) Zones may or may not be connected inside the reservoir; **preferential paths** may exist.

Tasks: (1) Determine and quantify these interactions!

(2) Construct production models which take these interactions into account and correctly allocate the production to the different inflow valves.

(3) Derive long-term production forecasts and suitable production strategies. Improve the **Ultimate Recovery**!

Hypothesis: Assume that suitable production polynomials p_A, p_B, and p_{AB} have been constructed.

Subideal Border Bases

Definition 3.1 Suppose we are given a set of polynomials $F = \{f_1, \ldots, f_m\} \subset P$ and order ideals $\mathcal{O}_1, \ldots, \mathcal{O}_m$.

(a) The set $\mathcal{O} = \mathcal{O}_1 f_1 + \cdots + \mathcal{O}_m f_m$ is called an **F-order ideal**.

(b) The set $\partial\mathcal{O}_F = (x_1\mathcal{O}_F \cup \cdots \cup x_n\mathcal{O}_F) \setminus \mathcal{O}_F$ is the **border** of $\mathcal{O}_F$.

(c) Let $\partial\mathcal{O}_F = \{b_1 f_{\beta_1}, \ldots, b_\nu f_{\beta_\nu}\}$. An **$\mathcal{O}_F$-subideal border prebasis** is a set $G = \{g_1, \ldots, g_\nu\}$ such that

$$g_j = b_j f_{\beta_j} - \sum_{i=1}^{\mu} c_{ij} t_i f_{\alpha_i}$$

(d) An $\mathcal{O}_F$-subideal border prebasis is called an **$\mathcal{O}_F$-subideal border basis** if the elements of $\mathcal{O}_F$ are a $\mathbb{R}$-basis of $J/(\langle G \rangle \cap J)$ for $J = \langle F \rangle$.

The Subideal Version of the AVI-Algorithm

Let $\mathbb{X} \subset [-1,1]^n$ be a finite set of points, let $\varepsilon > 0$ be a given threshold number, and let $F = \{f_1, \ldots, f_m\}$ be a set of non-zero unitary polynomials.

Then there exists an algorithm which computes an F-order ideal $\mathcal{O}_F$ and an **ε-approximate $\mathcal{O}_F$-subideal border basis** G which vanishes ε-approximately at $\mathbb{X}$.

Solution of the Production Allocation Problem

Let us define the **contributions** c_A and c_B to be the part of the total production passing through the corresponding down-hole valve.

Let x_A, x_B represent the valve positions. Here $x_i = 0$ means that the valve is closed and $x_i = 1$ corresponds to a fully opened valve.

If valve A is closed, i.e. for points in $\mathcal{Z}(x_A)$, there is no contribution from zone A. By **Hilbert's Nullstellensatz**, this means $p_A \in \langle x_A \rangle$ and similarly $p_B \in \langle x_B \rangle$.

To model p_{AB}, we write $p_{AB} = p_A + p_B + q_{AB}$ where q_{AB} is a polynomial which measures the **interaction** of the two zones.

To compute q_{AB}, we write $q_{AB} = f_A \cdot (x_B\, p_A) + f_B \cdot (x_A\, p_B)$ and note that this can be computed via the subideal version of the AVI-algorithm.

The result is $p_{AB} = p_A + p_B + f_A x_B p_A + f_B x_A p_B$ and satisfies $x_A = 0 \Rightarrow p_{AB} = p_B$ as well as $x_B = 0 \Rightarrow p_{AB} = p_A$.

The **contributions** of the two zone are then given by $c_A = (1 + f_A x_B) p_A$ and $c_B = (1 + f_B x_A) p_B$.

At the same time we gain a detailed insight into the nature of the **interactions** by examining the structure of the polynomials f_A, f_B.

4 – The Liquid-Gas Relationship

I went on a diet,
swore off drinking and heavy eating,
and in fourteen days, I lost two weeks.
(Joe E. Lewis)

Goal: In a well, compute model polynomials p_L and p_G for the liquid and the gas production. Then find **simple** relations

$$f_1 \cdot p_L + f_2 \cdot p_G = g$$

where the **remainder** g should be a **small** polynomial. Interpret f_1, f_2, g physically.

Mathematical Formulation: Find a pair (f_1, f_2) which is an **approximate syzygy** of (p_L, p_G).

Rational Approximation of Border Bases

Goal: Given an approximate border basis G, find a **nearby** exact border basis $\widetilde{G}$, i.e. an exact border basis in $\mathbb{Q}[x_1, \ldots, x_n]$ such that $G - \widetilde{G}$ consists of **small** polynomials.

Algorithm 1: Form the **approximate multiplication matrices** corresponding to G, compute their (approximate) solutions, and recompute the exact vanishing ideal of those solution points.

Algorithm 2: From the approximate multiplication matrices, compute a **family of commuting matrices** approximating it. Then read off the exact border basis.

Example 4.1 **(Zone A)**

Let us model Zone A of a certain two-zone well using the AVI algorithm. We use only the **driving** quantities

$$
\begin{aligned}
x[1]: &\quad xA(valve\ opening) \\
x[2]: &\quad \Delta P downstream \\
x[3]: &\quad \Delta P choke \\
x[4]: &\quad \Delta P transport\ tubing \\
x[5]: &\quad \Delta P inflow\ A
\end{aligned}
$$

After normalization to $[0, 1]$, the **fluid production** QA and the **gas production** GA are approximated (using $\varepsilon = 0.25$, mean evaluation error 0.12) by the following polynomials:

$$
\begin{aligned}
p_{QA} &\approx -0.55x_1x_4 + 1.64x_4x_5 - 0.03x_1 - 1.19x_3 - 0.98x_4 \\
&\quad +1.08\,(x_5-1)^2 + 0.62 \\
p_{GA} &\approx 0.01x_1x_4 + 0.84x_4x_5 - 0.01x_1 - 0.44x_3 - 0.54x_4 \\
&\quad +0.46\,(x_5-1)^2 + 0.37
\end{aligned}
$$

As we can see, we have $p_{QA} - 2.33\,p_{GA} \approx h(x_1,\ldots,x_5)$ with a "small" polynomial h that is essentially inversely correlated to x_4, the transport tubing pressure difference.

This results suggests the following interpretation: **Fluid and gas** production in Zone A are essentially **proportional**. However, at increasing production rates the **large gas volume in the production tubing** **reduces** **the fluid production**.

Example 4.2 **(Zone B)**

Now let us apply this procedure to Zone B of the same well. For x_5 we use the pressure difference between the annulus of zone B and the tubing pressure at zone A, i.e. the sum of $\Delta P\, production\; tubing\; BA$ and $\Delta P inflow\; B$. We find

$$\begin{aligned} p_{QB} &\approx 0.08x_1 + 2.97x_4 + 0.34x_5 - 2.13 \\ p_{GB} &\approx -0.16x_1 + 3.13x_4 + 0.28x_5 - 2.15 \end{aligned}$$

where the constant clearly corresponds to the shifting of x_4 we have performed. (The initial range was 13900 – 18600.) In both computations we used $\varepsilon = 0.25$ again. The mean evaluation error is 0.07 in each case.

The interpretation of this result is that QB and GB are essentially proportional. Since $p_{QB} - p_{GB} \approx 0.25\, x_1 - 0.15 x_4$, we see that **opening valve B favors fluid production** and the inhibiting effect of larger gas volumes in the production tubing is not as significant as for zone A.

Remark 4.3 **(General Liquid-Gas Relationship)**

(a) Model the zone/well using all driving forces, the liquid and the gas production. Let $\widetilde{G}$ be the computed approximate border basis.

(b) Using Rational Approximation, find an exact border basis G near $\widetilde{G}$.

(c) Compute a subideal border basis H for the driving forces of the zone/well which is contained in the ideal $\langle G \rangle$.

(d) The residue class ideal generated by G in $P/\langle H \rangle$ is the **set of liquid-gas relationships**. Interpret its generators.

5 – Further Applications of the AVI Algorithm

It's kind of fun to do the impossible.
(Walt Disney)

(1)(Exploration) Direct computation of algebraic relations in seismic shot data; algebraic determination of a velocity-like field;

(2)(Production-Exploration) Reconstruction of shape changes in a reservoir; relations between shape changes and production;

(3)(Transients) Dynamic version of the AVI algorithm using **differential polynomials**; modeling of time-dependent phenomena such as transients.

Applications to Other Industries

(1) **(Steel Industry)** Control of the heat function in an annealing process; modeling in the absence of physical laws;

(2) **(Financial Industry)** Modeling of the volatiliy parameter in **option price theory**; prediction of short and long term volatility functions;

(3) **(Heat Engineering)** Modeling of the heating and cooling systems at Passau University; control in highly non-linear dynamical systems;

(4) **(Optics)** Modeling of optical systems at their limit of resolution.

6 – The Algebraic Oil Project

Being right too soon is socially unacceptable.

(Robert A. Heinlein)

Research $\Leftrightarrow$ $\Updownarrow$	**SIEP** *Exploratory* *Research*	$\Leftrightarrow$ **Implementation** $\Updownarrow$
Passau University *Symbolic Computation*		**RISC Software** *Hagenberg (A)*
	CoCoA Library *Genova Univ.* $\Rightarrow$	**ApCoCoA Library** *Passau University*

This is not **THE END**

or the beginning of the end,

but it is the end of the beginning.

(Winston Churchill)

Thank you for your attention!