Algebraic Oil
Overview and Perspectives

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Questions, Please!

1. **What Is Algebraic Oil?**
2. **Why Polynomials?**
3. **What Is Algebraic Modelling?**
4. **Does It Work?**
5. **How Does It Work?**
6. **What Is It Good for?**
7. **What's Next?**

1 – What Is Algebraic Oil?

I can't tell you the secret of the universe,
because then it is no secret anymore.
(Mullah Nasruddin)

The basic idea is to apply methods from computational **Algebra** to problems in **Oil** production and exploration.

Computational Mathematics

Symbolic Computation

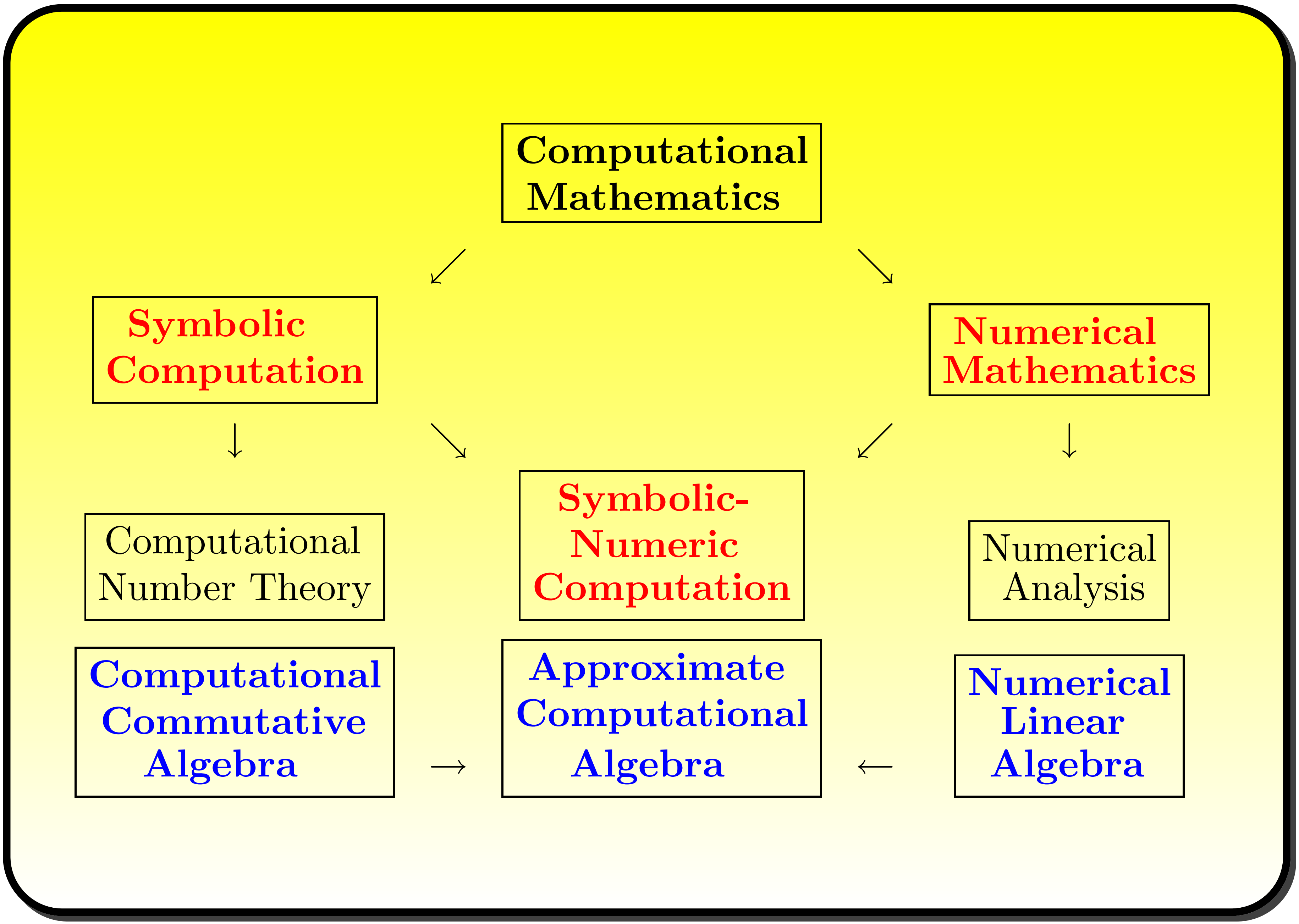
Computational Number Theory

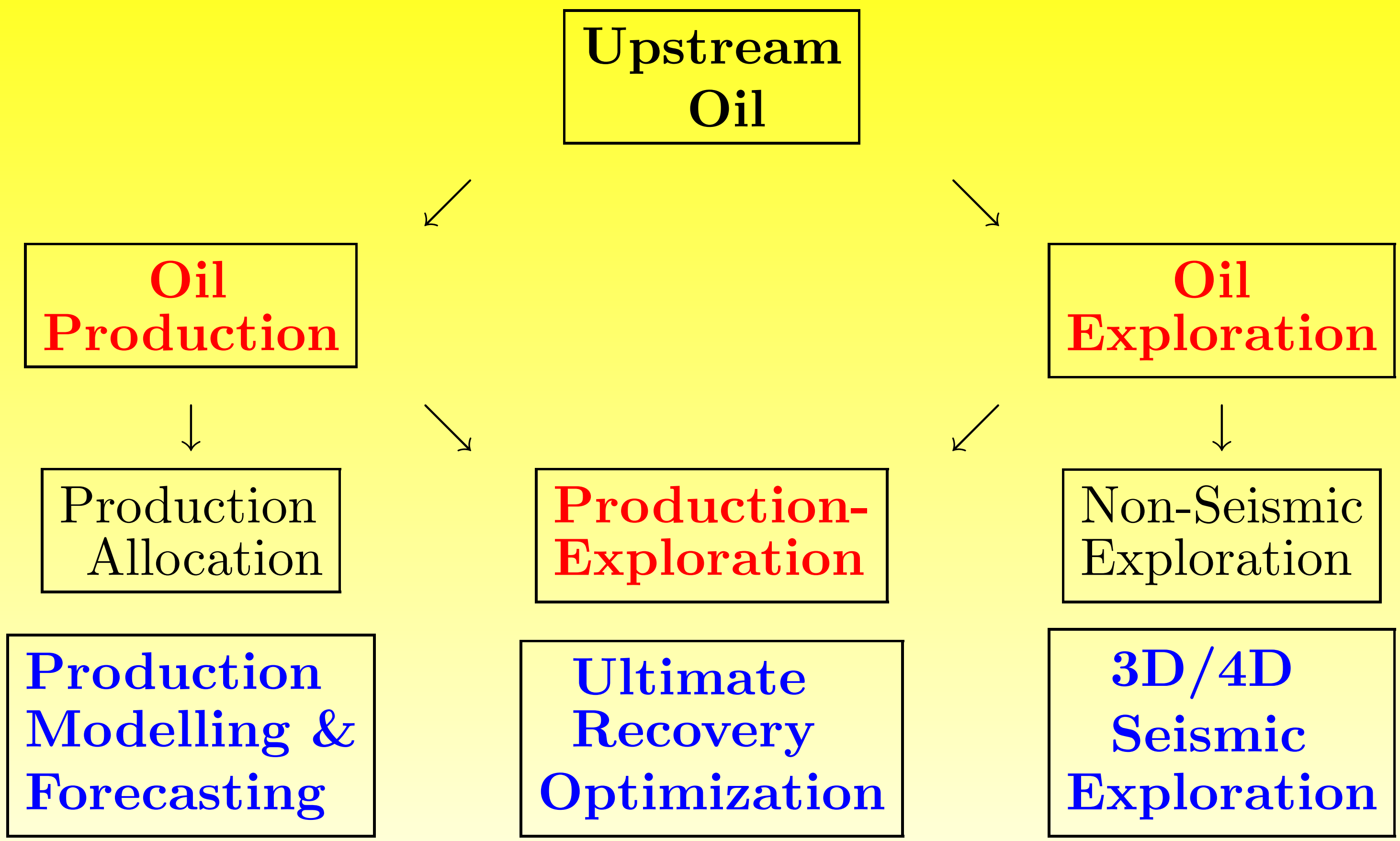
Computational Commutative Algebra

Numerical Mathematics

Numerical Analysis

Numerical Linear Algebra


Computational Mathematics
Symbolic Computation
Numerical Mathematics
Computational Number Theory
Symbolic-Numeric Computation
Numerical Analysis
Computational Commutative Algebra
Approximate Computational Algebra
Numerical Linear Algebra


Upstream Oil
Oil Production
Oil Exploration
Production Allocation
Production-Exploration
Non-Seismic Exploration
Production Modelling & Forecasting
Ultimate Recovery Optimization
3D/4D Seismic Exploration

To a man with a hammer
everything looks like a nail.

Basic Tenet 1

Use the mathematical theory which is most suitable to your industrial problem.

Don't let your pre-knowledge determine the tools you use. Let the application decide which theory works best. If necessary, widen your mathematical scope.

Basic Tenet 2

Let the application steer your research.

Let it decide which mathematical problem is the next one you should study.

2 – Why Polynomials?

Before you criticize someone,
you should walk a mile in their shoes.
That way, when you criticize them,
you're a mile away and you have their shoes.

$\mathbb{R}$ field of real numbers

$x_1, \dots, x_n$ **indeterminates** (variables, unknowns)

$x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}$ **term** in the indeterminates $x_1, \dots, x_n$ (where $\alpha \geq 0$)

$f = c_1 t_1 + \cdots + c_s t_s$ **polynomial** in the indeterminates $x_1, \dots, x_n$ (where $c_i \in \mathbb{R}$ and t_i term)

$P = \mathbb{R}[x_1, \dots, x_n]$ **polynomial ring**

Basic modelling problem: Suppose we are given finitely many data points $p_1, \ldots, p_s \in \mathbb{R}^n$ and values $a_1, \ldots, a_s \in \mathbb{R}$ of a function f. Find a **simple** function f such that $f(p_i) = a_i$ for $i = 1, \ldots, s$.

Reasons for using polynomials:

• On a finite data set, every function is polynomial.

• Polynomials are dense in many spaces of functions.

• Many important physical laws are given by polynomials (e.g. $F - ma = 0$, $r^2F - \gamma m_1 m_2$, $E - mc^2 = 0$, $\ldots$).

• Polynomials of bounded degree form a sufficiently **rigid** interpolation space to offer enough **predictive power**.

3 – What Is Algebraic Modelling?

It is kind of fun to do the impossible.
(Walt Disney)

General Modelling Problem: A complex physical system (e.g. an oil production system) is to be modelled. We have at our disposal several time series of measurements (production data, test data) and certain partial differential equations describing particular aspects of the system.

Traditional Modelling Technique:

- Write down the partial differential equations locally using indeterminate coefficients.

• Using the measured data (and *a priori* assumptions), try to fit as many coefficients as closely as possible.

• Solve the resulting system of PDEs using standard numerical analysis techniques.

Problems:

• The measured data may not suffice to determine **all** coefficients in the PDEs, since there are too many.

• There may not be a reasonably **simple** function solving all PDEs.

• The interpolation space is too big (splines, analytic functions). The predictive power of the model is low: for every future development, there is a model compatible with the measured data and that future development.

Algebraic Modelling Technique

- Starting from the measured data only, use the ABM- or AVI-algorithms to construct **simple** polynomials which model the system within a specified tolerance.
- Use only **obvious** physical knowledge to guide the computation.

Basic Tenet 3

Do not impose *a priori* model assumptions on a physical system. As far as possible, rely only on measured data. Try to recover basic physical laws (e.g. Bernoulli's Law) instead of imposing them.

4 – Does It Work?

Weather forecast for tonight:

DARK.

Suppose we are given an oil production system (called CW29 here) and several time series of measurements:

x_1 valve position for zone A (0-100)

x_2 valve position for zone B (0-100)

$x_3 = \Delta P_{choke}$ flow pressure change at downstream choke (0-5000)

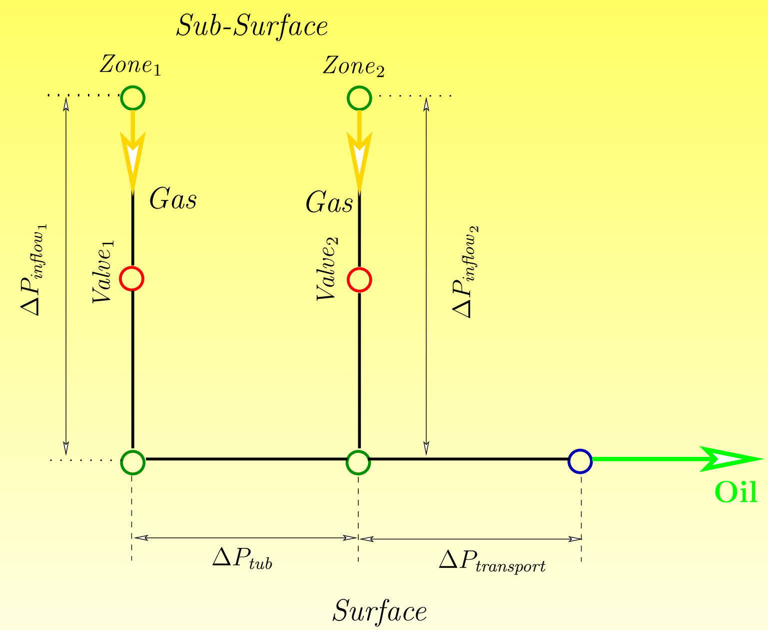
$x_4 = \Delta P_{prod.\ tubing}$ pressure change inflow B $\rightarrow$ inflow A (0-3010)

$x_5 = \Delta P_{inflowB}$ reservoir pressure $\rightarrow$ tubing pressure at B (0-7000)

$x_6 = \Delta P_{downstr.}$ downstream pressure choke $\rightarrow$ manifold (9-216)

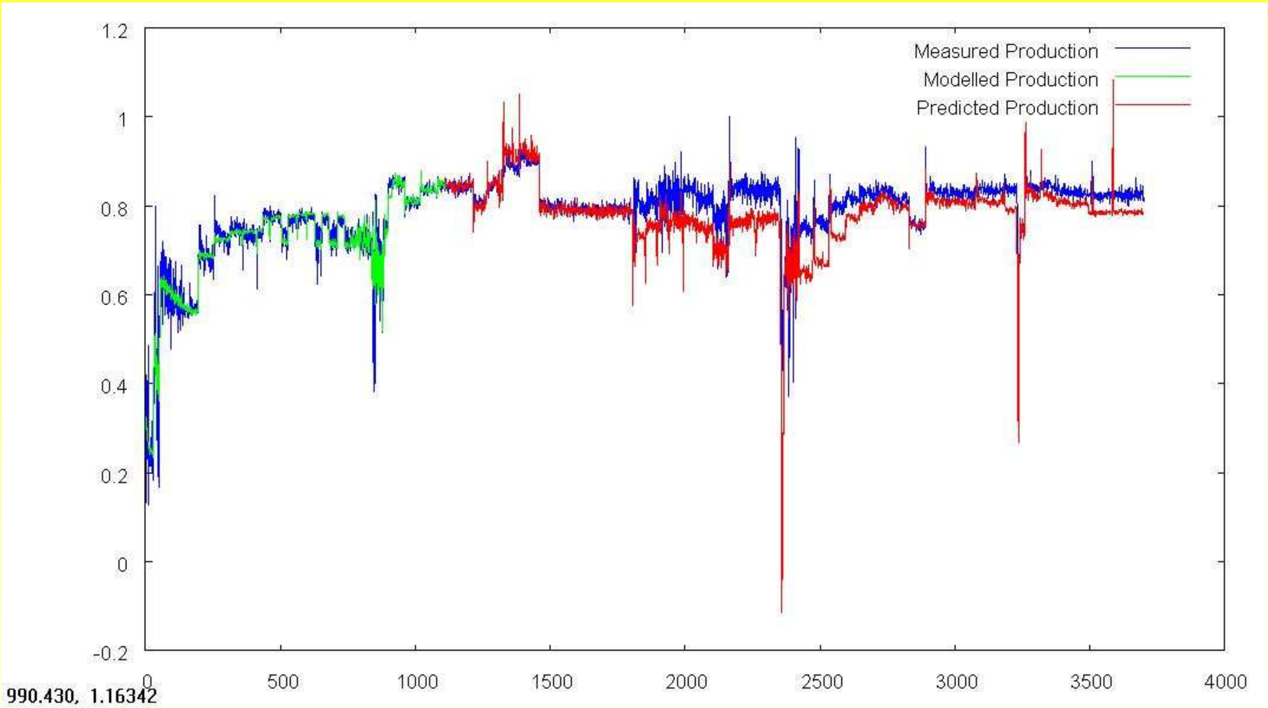
$x_7 = \Delta P_{inflow A}$ reservoir pressure $\rightarrow$ tubing pressure at A (0-1805)

$x_8 = \Delta P_{transp.\ tubing}$ pressure change inflow A $\rightarrow$ tubing head (THP) (5300-10550)



Algebraic Modelling using the AVI-Algorithm, threshold $\epsilon = 0.15$

30% of data used for modelling, 70% used for checking the prediction



5 – How Does It Work?

All models are wrong.
Some models are useful.
(George Box)

Exact Modelling Problem: Given finitely many points $p_1, \ldots, p_s \in \mathbb{R}^n$ and numbers $a_1, \ldots, a_s \in \mathbb{R}^n$, find a polynomial **model** $f \in \mathbb{R}[x_1, \ldots, x_n]$ such that $f(p_i) = a_i$ for $i = 1, \ldots, s$.

Thus we are asking for **multivariate interpolation** using the **polynomial ring** $P = \mathbb{R}[x_1, \ldots, x_n]$.

Remark. The polynomial f is not unique. Two such polynomials differ by an element of the set

$$I = \{g \in \mathbb{R}[x_1, \ldots, x_n] \mid g(p_1) = \cdots = g(p_s) = 0\}$$

This set is called the **vanishing ideal** of $\mathbb{X} = \{p_1, \ldots, p_s\}$. It is a polynomial ideal, i.e. we have $I + I \subseteq I$ and $P \cdot I \subseteq I$.

Approximate Interpolation Problem: Given a bound $\epsilon > 0$, find a polynomial f as above such that $|f(p_i) - a_i| < \epsilon$ for $i = 1, \ldots, s$.

Problem: All polynomials having sufficiently small coefficients vanish approximately at $\mathbb{X}$.

The set of polynomials which vanish approximately at $\mathbb{X}$ is **not** a polynomial ideal.

We need to measure the **size** of a polynomial.

Definition. Given a polynomial $f = c_1 t_1 + \cdots + c_s t_s$ with $c_i \in \mathbb{R}$ and terms $t_i \in \mathbb{T}^n$, we let

$$\|f\| = \|(c_1, \ldots, c_s)\|$$

be the (euclidean) **norm** of the coefficient vector of f. This defines a norm on the vector space P.

Definition. **(a)** Given a bound $\epsilon > 0$, we say that a polynomial $f \in P$ **vanishes ϵ-approximately** at $\mathbb{X}$ if $|f(p_i)| < \epsilon \cdot \|f\|$ for $i = 1, \ldots, s$.

(b) A polynomial ideal I which is generated by unitary polynomials $f_1, \ldots, f_m$ (i.e. by polynomials f_j satisfying $\|f_j\| = 1$) which vanish ϵ-approximately at $\mathbb{X}$ is called an **ϵ-approximate vanishing ideal** of $\mathbb{X}$.

Border Bases

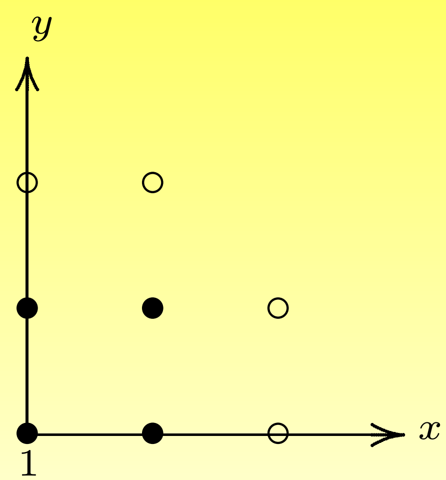
When they started to prove even the simplest claims, many turned out to be wrong.
(Bertrand Russell)

Definition. (a) A finite set of terms $\mathcal{O}$ is called an **order ideal** if $t \in \mathcal{O}$ and $t' \mid t$ imply $t' \in \mathcal{O}$.

(b) Let $\mathcal{O}$ be an order ideal. The set $\partial\mathcal{O} = (x_1\mathcal{O} \cup \cdots \cup x_n\mathcal{O}) \setminus \mathcal{O}$ is called the **border** of $\mathcal{O}$.

(c) A polynomial ideal I has an **$\mathcal{O}$-border basis** if the terms in $\mathcal{O}$ form a vector space basis of P/I. (Equivalently, we want $P = I \oplus \langle \mathcal{O} \rangle_K$.)

A Picture of an Order Ideal and its Border



Properties of Border Bases

Let $\mathcal{O} = \{t_1, \ldots, t_\mu\}$ be an order ideal and $\partial\mathcal{O} = \{b_1, \ldots, b_\nu\}$ its border.

Property 1. If an ideal I has an $\mathcal{O}$-border basis, it is generated by (uniquely determined) polynomials of the form

$$g_j = b_j - c_{1j}t_1 - \cdots - c_{\mu j}t_\mu$$

where $c_{ij} \in \mathbb{R}$ and $j = 1, \ldots, \nu$. These polynomials are called the **$\mathcal{O}$-border basis** of I.

Property 2. Given any set of polynomials $G = \{g_1, \ldots, g_\nu\}$ of this form (i.e. a so-called **border prebasis**) there are uniquely determined equations $\mathrm{NR}_G(S_{ij}) = 0$ for the coefficients c_{ij} that have to be satisfied in order for G to be a border basis.

Property 3. Border bases are **numerically stable**. Small changes in the coordinates of the points lead to small changes in the coefficients c_{ij}. Small changes in the coefficients c_{ij} (preserving the equations) keep the property of being a border basis.

Property 4. Border bases frequently keep **symmetries**.

Property 5. Border bases provide an explicit parametrization of all 0-dimensional polynomial ideals.

Approximate Border Bases

I had a fortune cookie and it said:

"Outlook not so good!"

"Sure, but Microsoft ships it anyway!"

Definition. Let $\mathcal{O} = \{t_1, \dots, t_\mu\}$ be an order ideal and $\partial\mathcal{O} = \{b_1, \dots, b_\nu\}$ its border. Let $\epsilon > 0$.

A set of polynomials $G = \{g_1, \dots, g_\nu\}$ is called an **ϵ-approximate $\mathcal{O}$-border basis** if

(a) it is an $\mathcal{O}$-border prebasis, i.e. $g_j = b_j - c_{1j}t_1 - \cdots - c_{\mu j}t_\mu$ with $c_{ij} \in \mathbb{R}$ for $j = 1, \dots, \nu$, and

(b) the equations defining border bases are **almost** satisfied, i.e. we have $|\mathrm{NR}_G(S_{ij})| < \epsilon$ for all i, j.

The AVI-Algorithm

What gets us into trouble is not what we don't know, its what we know for sure that just ain't so.

(Mark Twain)

Goal: Given a set of (approximate) points $\mathbb{X} = \{p_1, \ldots, p_s\}$ in $\mathbb{R}^n$ and $\epsilon > 0$, find an order ideal $\mathcal{O}$ and an approximate $\mathcal{O}$-border basis G such that the polynomials in G vanish ϵ-approximately at the points of $\mathbb{X}$.

Notice that, in general,

- we have $\#\mathcal{O} << \#\mathbb{X}$,
- the ideal $\langle G \rangle$ is the unit ideal.

Theorem 5.1 **(The AVI Algorithm)**
The following algorithm computes an approximate border basis of an approximate vanishing ideal of a finite set of points $\mathbb{X} \subseteq \mathbf{[-1,1]^n}$.

A1 *Start with lists* $G = \emptyset$, $\mathcal{O} = [1]$, *a matrix* $\mathcal{M} = (1, \dots, 1)^{\mathrm{tr}} \in \mathrm{Mat}_{s,1}(\mathbb{R})$, *and* $d = 0$.

A2 *Increase* d *by one and let* L *be the list of all terms of degree* d *in* $\partial\mathcal{O}$, *ordered decreasingly w.r.t.* σ. *If* $L = \emptyset$, *return the pair* $(G, \mathcal{O})$ *and stop. Otherwise, let* $L = (t_1, \dots, t_\ell)$.

A3 *Let* m *be the number of columns of* $\mathcal{M}$. *Form the matrix*

$$\mathcal{A} = (\mathrm{eval}(t_1), \dots, \mathrm{eval}(t_\ell), \mathcal{M}) \in \mathrm{Mat}_{s,\ell+m}(\mathbb{R}).$$

Using its **SVD**, *calculate a matrix* $\mathcal{B}$ *whose column vectors are an ONB of the* **approximate kernel** $\mathrm{apker}(\mathcal{A}, \epsilon)$.

A4 Compute the **stabilized reduced row echelon form** of $\mathcal{B}^{\mathrm{tr}}$ with respect to the given τ. The result is a matrix $\mathcal{C} = (c_{ij}) \in \mathrm{Mat}_{k,\ell+m}(\mathbb{R})$ such that $c_{ij} = 0$ for $j < \nu(i)$. Here $\nu(i)$ denotes the column index of the pivot element in the i^{th} row of $\mathcal{C}$.

A5 For all $j \in \{1, \ldots, \ell\}$ such that there exists a $i \in \{1, \ldots, k\}$ with $\nu(i) = j$ (i.e. **for the column indices of the pivot elements**), append the polynomial

$$c_{ij} t_j + \textstyle\sum_{j'=j+1}^{\ell} c_{ij'} t_{j'} + \sum_{j'=\ell+1}^{\ell+m} c_{ij'} u_{j'}$$

to the list G, where $u_{j'}$ is the $(j' - \ell)^{\mathrm{th}}$ element of $\mathcal{O}$.

A6 For all $j = \ell, \ell - 1, \ldots, 1$ such that the j^{th} column of $\mathcal{C}$ contains **no pivot element**, append the term t_j as a new first element to $\mathcal{O}$ and append the column $\mathrm{eval}(t_j)$ as a new first column to $\mathcal{M}$.

A7 Using the SVD of $\mathcal{M}$, calculate a matrix $\mathcal{B}$ whose column vectors are an ONB of $\operatorname{apker}(\mathcal{M}, \epsilon)$.

A8 Repeat steps **A4** – **A7** until $\mathcal{B}$ is empty. Then continue with step **A2**.

This algorithm returns the following results:

(a) The set $\mathcal{O} = \{t_1, \ldots, t_\mu\}$ is an order ideal of terms which is **strongly linearly independent** on $\mathbb{X}$, i.e. such that there is no unitary polynomial in $\langle \mathcal{O} \rangle_K$ which vanishes ϵ-approximately on $\mathbb{X}$.

(b) The set G is a **δ-approximate $\mathcal{O}$-border basis**. (An explicit bound for δ can be given.)

Modelling Oil Production

Finagle's First Law:
If an experiment works,
something has gone wrong.

Example 5.2 For a certain oil well, we have 7400 data points in $\mathbb{R}^8$. We use 30% of the points for modelling the total production in the following way:

(a) Using the AVI-Algorithm with $\epsilon = 0.1$, compute an order ideal $\mathcal{O}$ and its evaluation matrix $\mathrm{eval}(\mathcal{O})$.

(b) Find the vector in the linear span of the rows of $\mathrm{eval}(\mathcal{O})$ which is closest to the total production.

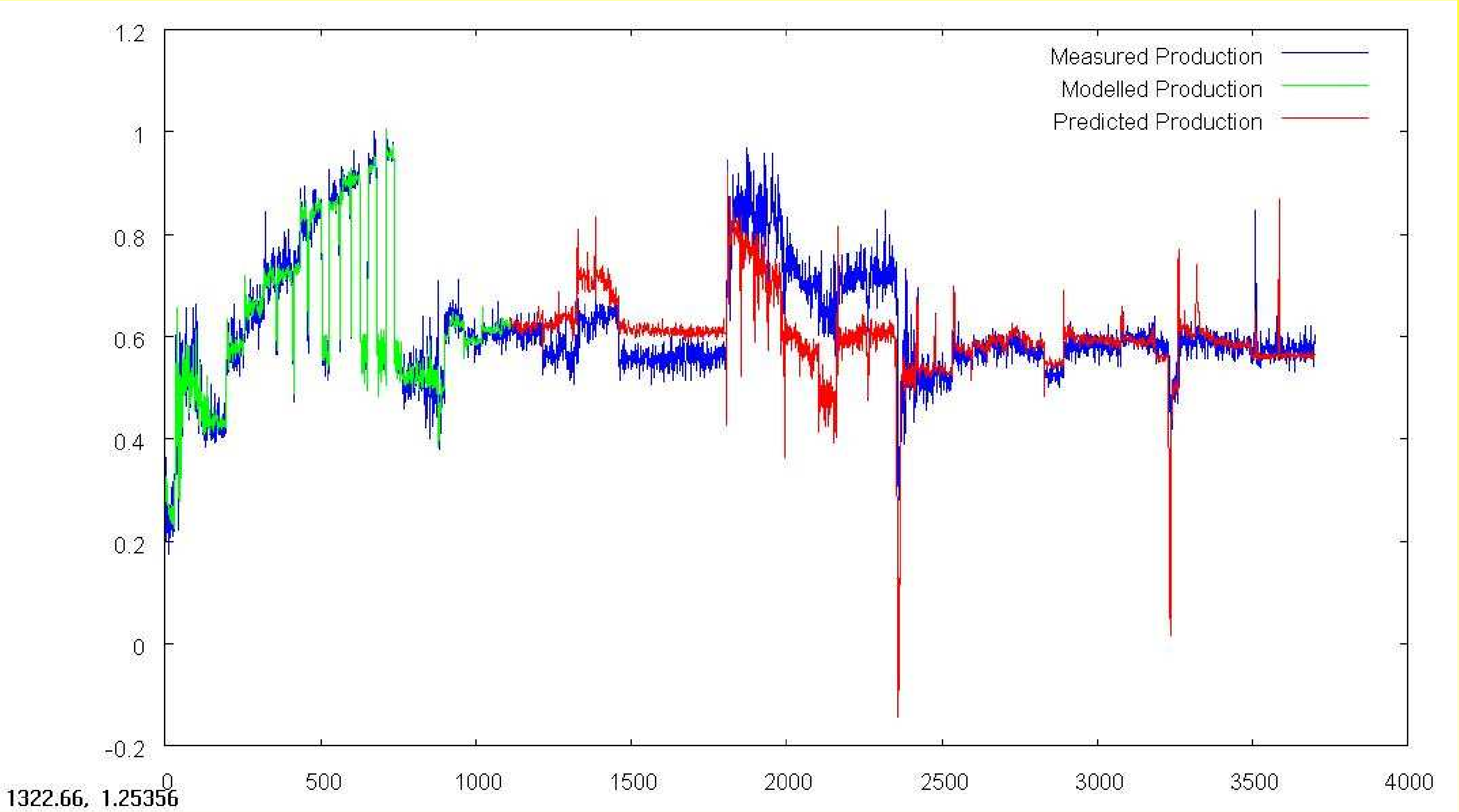
(c) The corresponding linear combination of terms in $\mathcal{O}$ is the **model polynomial** for the total production.

(d) Plot all values of the model polynomial at the given points. Compare them at the points which were not used for modelling with the actual measured values.

Example 5.3 Using the same data points, $\epsilon = 0.1$, and the same procedure, model the total gas production.

Algebraic Modelling of CW29 gas production, AVI-Alg., $\epsilon = 0.1$

30% of data used for modelling, 70% used for checking the prediction



6 – What Is It Good for?

These are the conclusions
on which I base my facts.
(Adlai Stevenson)

Applications of Algebraic Modelling Techniques:

- Long-term production forecasts
- Subsurface interactions between production zones
- Production allocation

Long Term Production Forecasts

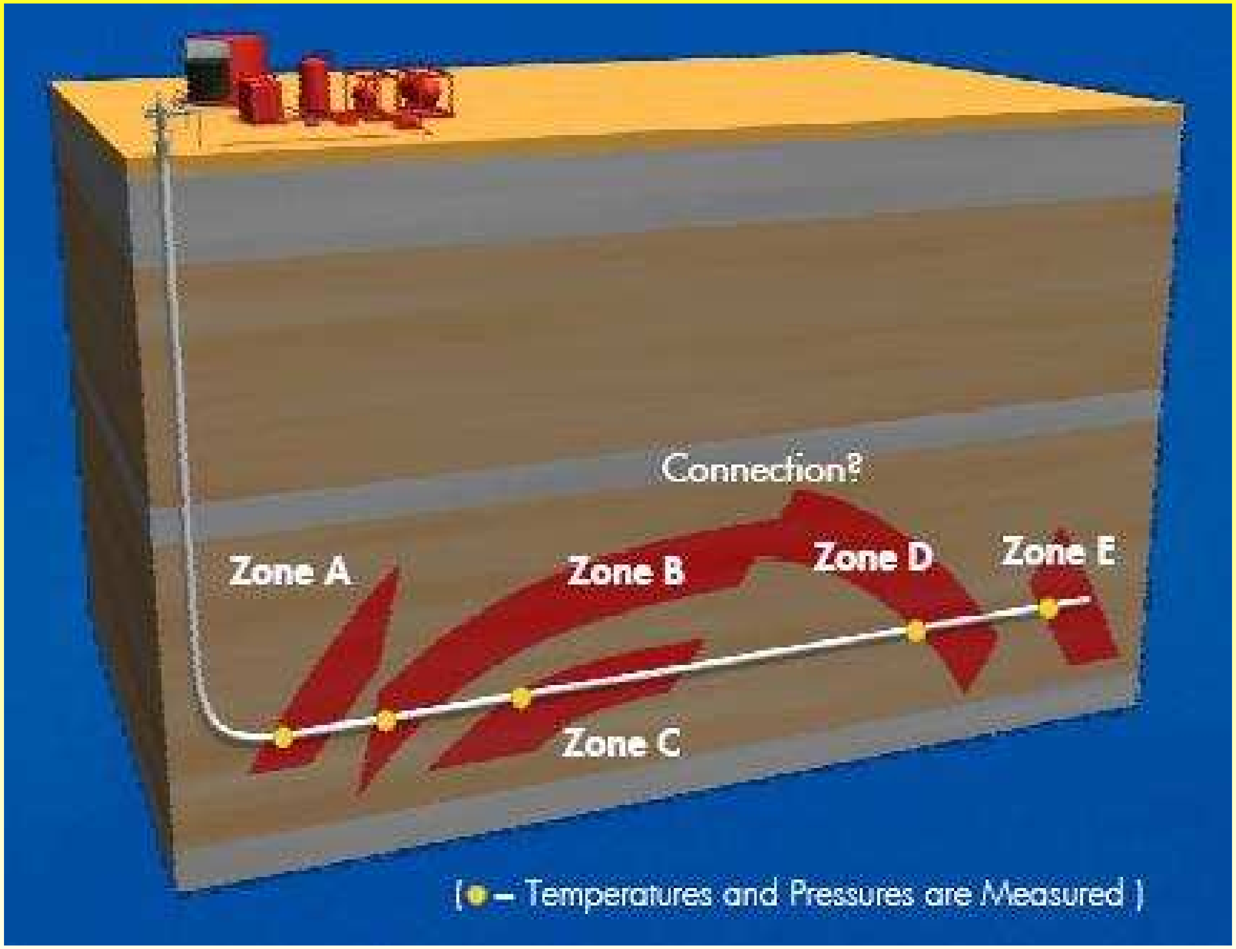
- Produce production models for longer time scales:

$$\text{hours} \rightarrow \text{days} \rightarrow \text{weeks} \rightarrow \text{months}$$

- Study the changes of the coefficients of the production polynomials over time

- Predict discontinuities (gassing out, water breakthrough)

Subsurface Interactions



Interpretation of the production polynomial:

Consider the indeterminates x_i as **random variables** which assume "random" values at the different points.

Then the coefficients of the indeterminates in the production polynomial are the **main effects**.

The coefficients of the terms $x_i x_j$ represent the **correlation coefficients** of the quantities corresponding to x_i and x_j.

Thus these coefficients measure the strength of the **interaction** between x_i and x_j. Their sign determines whether this interaction yields a positive or negative contribution to the production.

Example 6.1 **(Fluid and Gas Interaction)**

Let us model **Zone A** of a certain two-zone well using the AVI algorithm. We use only the **driving** quantities

$$
\begin{aligned}
x[1]: &\quad xA\ (valve\ opening) \\
x[2]: &\quad \Delta Pdownstream \\
x[3]: &\quad \Delta Pchoke \\
x[4]: &\quad \Delta Ptransport\ tubing \\
x[5]: &\quad \Delta Pinflow\ A
\end{aligned}
$$

After normalization to $[0,1]$, the **fluid production** QA and the **gas production** GA are approximated (using $\epsilon = 0.25$, mean evaluation error 0.12) by the following polynomials:

$$
\begin{aligned}
p_{QA} &\approx -0.55x_1x_4 + 1.64x_4x_5 - 0.03x_1 - 1.19x_3 - 0.98x_4 \\
&\quad +1.08\,(x_5-1)^2 + 0.62 \\
p_{GA} &\approx 0.01x_1x_4 + 0.84x_4x_5 - 0.01x_1 - 0.44x_3 - 0.54x_4 \\
&\quad +0.46\,(x_5-1)^2 + 0.37
\end{aligned}
$$

As we can see, we have $p_{QA} - 2.33\,p_{GA} \approx h(x_1,\ldots,x_5)$ with a "small" polynomial h that is essentially inversely correlated to x_4, the transport tubing pressure difference.

This results suggests the following interpretation: Fluid and gas production in Zone A are essentially **proportional**. However, at increasing production rates the **large gas volume** in the production tubing **reduces the fluid production**.

Example 6.2 **(Gas Interacting with Oil Production)**

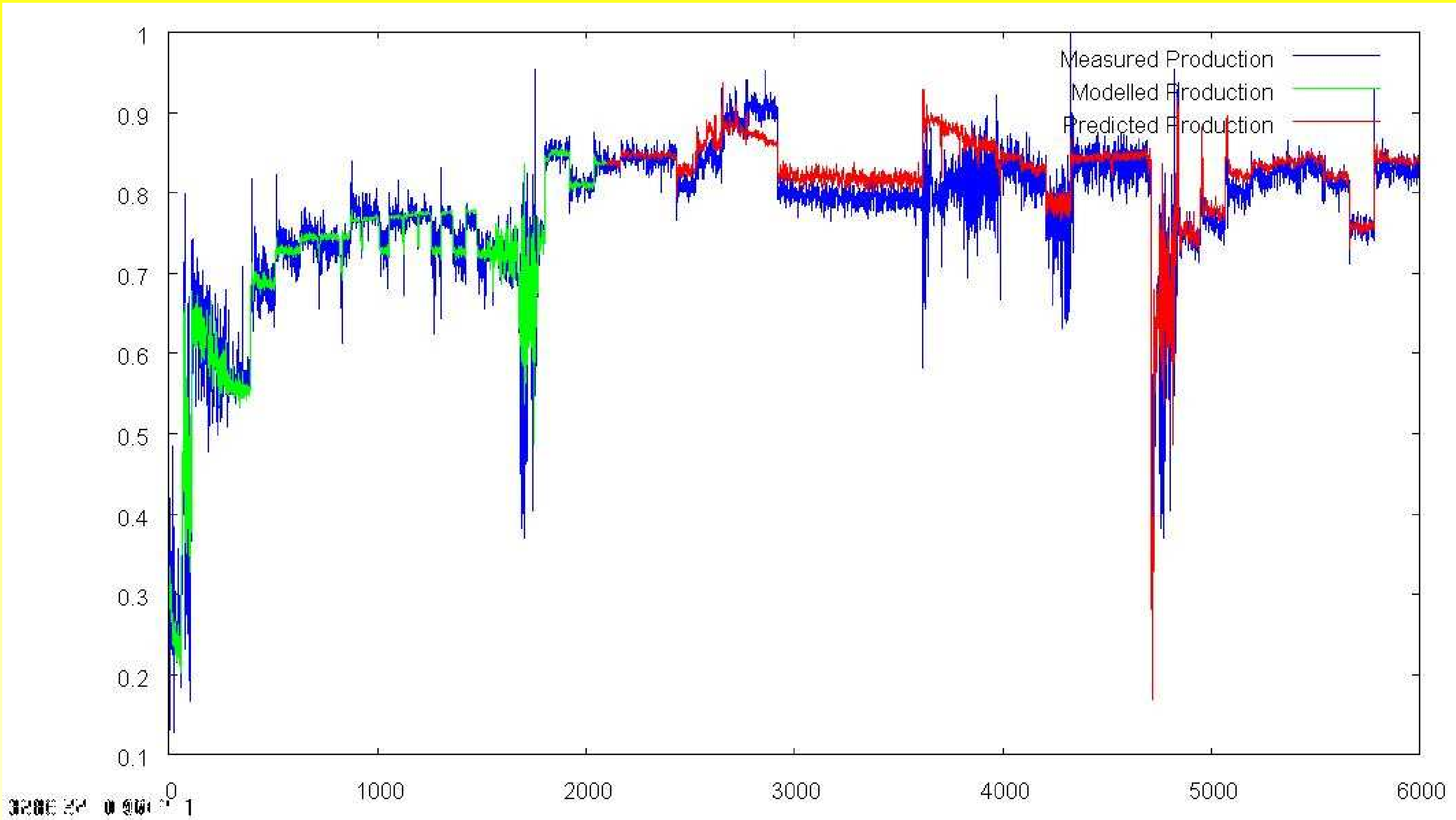
Now we model the oil production of zones A/B of well CW29 using the following quantities:

$$\begin{aligned} x[1] &\quad : \quad \Delta P_{valve\,A} \\ x[2] &\quad : \quad \Delta P_{valve\,B} \\ x[3] &\quad : \quad Gas\ Production \\ x[4] &\quad : \quad \Delta P_{prod.tubing} \\ x[5] &\quad : \quad \Delta P_{transp.tubing} \end{aligned}$$

We use 35% of the 6000 data points and $\epsilon = 0.1$.

The computed order ideal has 12 terms:

$$\{1, x_5, x_4, x_3, x_2, x_1, x_5^2, x_4x_5, x_3x_5, x_3x_4, x_1x_4, x_1x_3\}$$



Measured Production
Modelled Production
Predicted Production
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1000
2000
3000
4000
5000
6000

The oil production polynomial is approximately given by

$$0.14\,x_1x_3 - 0.23\,x_1x_4 + 0.39\,x_3x_4 - 1.43\,x_3x_5 + 1.2\,x_4x_5 - 0.3\,x_5^2$$
$$+0.11\,x_1 + 1.46\,x_3 - 1.25\,x_4 + 1.6x_5 \; - 0.55$$

Its terms involving x_3 (gas production) may be interpreted as follows:

$0.14\,x_1x_3$	gas is mainly produced at valve A (no term x_2x_3)
$1.46\,x_3$	gas stimulates oil production
$-1.43\,x_3x_5$	with increasing production, gas inhibits oil
$0.39\,x_3x_4$	to be ignored, since x_4 takes very small values

The Ultimate Goal: Increase the **ultimate recovery** by constructing a long term model and adjusting the production strategy according to its predictions.

Further Results: Determine a model for the liquid-gas relationship, adjust the production parameters to achieve an optimal liquid-gas relationship.

Production Allocation

When working towards the solution of a problem it helps if you know the answer.

Problem: Several zones of a well are producing together **(commingled production)**. Some of them are influencing each other. Typical interactions are:

(1) The oil produced by one zone may push back the oil trying to flow in at another zone.

(2) The gas which is produced simultaneously with the oil may have stimulating or inhibiting effects **(liquid-gas relationship)**.

(3) Zones may or may not be connected inside the reservoir; **preferential paths** may exist.

Tasks: (1) Determine and quantify these interactions!

(2) Construct production models which take these interactions into account and correctly allocate the production to the different inflow valves.

(3) Derive long-term production forecasts and suitable production strategies. Improve the **Ultimate Recovery**!

Hypothesis: Assume that suitable production polynomials p_A, p_B, and p_{AB} have been constructed during suitable well tests.

The Subideal Version of the AVI-Algorithm

Let $\mathbb{X} \subset [-1,1]^n$ be a finite set of points, let $\epsilon > 0$ be a given threshold number, and let $F = \{f_1, \ldots, f_m\}$ be a set of non-zero unitary polynomials.

Then there exists an algorithm which computes an F-order ideal $\mathcal{O}_F$ and an **ϵ-approximate $\mathcal{O}_F$-subideal border basis** G which vanishes ϵ-approximately at $\mathbb{X}$.

Using this algorithm, we can construct approximate interpolation polynomials which are contained in the given polynomial ideal $\langle f_1, \ldots, f_m \rangle$.

Solution of the Production Allocation Problem

Let us define the **contributions** c_A and c_B to be the part of the total production passing through the corresponding down-hole valve.

Let x_A, x_B represent the valve positions. Here $x_i = 0$ means that the valve is closed and $x_i = 1$ corresponds to a fully opened valve.

If valve A is closed, i.e. for points in $\mathcal{Z}(x_A)$, there is no contribution from zone A. By **Hilbert's Nullstellensatz**, this means $p_A \in \langle x_A \rangle$ and similarly $p_B \in \langle x_B \rangle$.

To model p_{AB}, we write $p_{AB} = p_A + p_B + q_{AB}$ where q_{AB} is a polynomial which measures the **interaction** of the two zones.

To compute q_{AB}, we write $q_{AB} = f_A \cdot (x_B\, p_A) + f_B \cdot (x_A\, p_B)$ and note that this can be computed via the subideal version of the AVI-algorithm.

The result is $p_{AB} = p_A + p_B + f_A x_B p_A + f_B x_A p_B$ and satisfies $x_A = 0 \Rightarrow p_{AB} = p_B$ as well as $x_B = 0 \Rightarrow p_{AB} = p_A$.

The **contributions** of the two zone are then given by $c_A = (1 + f_A x_B) p_A$ and $c_B = (1 + f_B x_A) p_B$.

At the same time we gain a detailed insight into the nature of the **interactions** by examining the structure of the polynomials f_A, f_B.

7 – What's Next?

Prediction is very difficult.
Especially if it's about the future.
(Niels Bohr)

The algebraic modelling techniques can be extended to cover more applications in the oil & gas industry:

- Dynamic modelling (using differential polynomials)
- Geometric exploration (3D algebraic modelling)
- Production-exploration (4D algebraic modelling)

Dynamic Modelling

Change is inevitable.
Except from a vending machine.

Idea: So far we have only produced **steady-state models**. If dynamic changes occur, we should take the time-derivatives of the physical quantities into account.

Algebraic Formulation: Instead of polynomials, use **differential polynomials**. Besides the indeterminates $x_1, \ldots, x_n$, they use indeterminates $\dot{x}_1, \ldots, \dot{x}_n$ for their (time-) derivatives, indeterminates $\ddot{x}_1, \ldots, \ddot{x}_n$ for their second (time-) derivatives, etc.

Problems: The **differential polynomial ring** $D = \mathbb{R}[x_1, \ldots, x_n, \dot{x}_1, \ldots, \dot{x}_n, \ddot{x}_1, \ldots, \ddot{x}_n, \ldots]$ is not noetherian, i.e. not every ideal is finitely generated.

EMR1: Approximate Differential Algebra

• Develop the Groebner basis theory for differential polynomials of **Bluhm-K.** further.

• Develop a differential analogue of border basis theory.

• Develop differential versions of the **BM-algorithm** for computing vanishing ideals of points.

• Develop and implement approximate versions of the differential BM-algorithm (**differential AVI-algorithm**)

• Apply the new algorithms to actual industrial data.

Applications of the DAVI-Algorithm

Modelling Transients: Operations such as starting up or shutting down a platform, switching a well from production to testing or vice versa, etc. introduce dynamic changes in the measured quantities; to model transients we need differential polynomials and the DAVI-algorithm.

Optimizing Transients: Modelling of the reaction of the production system with respect to control operations allows predictions. Derive optimal transients (e.g. start-up sequences) to avoid undesirable intermediate states (e.g. water breakthrough) and desirable final states (e.g. high production level).

Geometric Exploration

Your brain works faster than you think.

Idea: Traditional 3D seismic modelling involves substantial human intervention — it is like an art. Using a **3D algebraic modelling** technique, find algorithms to replace non-algorithmic steps.

Main Goal: Find a model for the shape of the oil/gas body which is an algebraic surface. In this way, identify also non-traditional shapes.

From **Robertson, Seismic Processing Tutorial**:

The picking of seismic velocities is one of the most boring, routine and **important** *jobs still left firmly in the hands of the processing geophysicist.*

The picking of velocities requires a very detailed knowledge of the geology.

3D Algebraic Modelling: • **Use the seismic shot data.**

(Traditional seismic modelling uses "seismic data" in which the shot data have undergone already some transformations that destroy geometric information.)

• Apply a suitable version of the ABM-algorithm to recover the **velocity field**.

• Using cut-off functions or level sets, construct a cloud of points that delineates approximately the border of the oil/gas body.

• Again use a suitable version of the AVI-algorithm to construct an algebraic surface that approximates this border.

Applications: • Algorithmic computation of velocity fields putting only moderate demands on **signal-to-noise ratios**.

• Discovery of oil/gas bodies having **non-traditional geometries**.

Production-Exploration

Being right too soon is socially unacceptable.

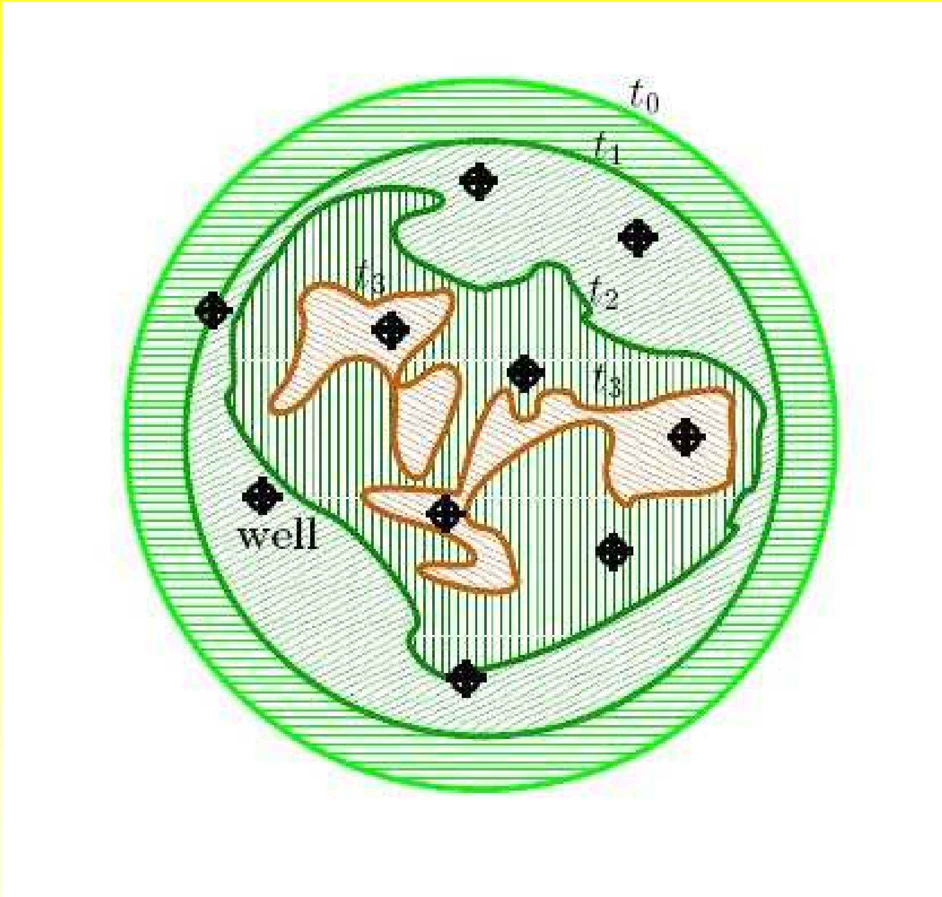
(Robert A. Heinlein)

Idea: Suppose that for a certain oil/gas body repeated seismic surveys are available. Using the **Geometric Exploration** technique (or a traditional modelling technique) construct algebraic surfaces which delineate the shapes of this oil/gas body at different points in time (**4D Algebraic Modelling**).

Application 1: Estimation of oil/gas bodies (appraisal phase).

Application 2: The change in shape is caused by production. Analysing the **deformation** of the algebraic surface, relate it to the production, and predict the further development under different production scenarios. Derive a production strategy which induces a desirable change in the geometry of the oil/gas body.

Schematic Example of Shape Changes



Applications in Other Areas

Your theory is crazy.
But it's not crazy enough to be true.
(Niels Bohr)

- **steel industry**: heat control during annealing
- **financial industry**: option price volatility modelling
- **optical industry**: camera calibration at the resolution limit
- **heat engineering**: heating and cooling systems at Passau university
- **banking industry**: modelling of sales department

This is not **THE END**

or the beginning of the end,

but it is the end of the beginning.

(Winston Churchill)

Thank you for your attention!

In the end, everything is a gag.

(Charlie Chaplin)