From seismic data to algebraic surfaces

Jan Limbeck

Department of Informatics and Mathematics
University of Passau

04/03/2010
Algebraic Oil Project Workshop, Passau

Outline

1. Motivation
   - General Setting
2. Timeline
3. Developement and implementation of algorithms
   - ABM algorithm
   - The extended ABM algorithm
4. Applications
   - Application of the extended ABM
   - Application of the ABM algorithm

Outline

1. Motivation
   - General Setting
2. Timeline
3. Developement and implementation of algorithms
   - ABM algorithm
   - The extended ABM algorithm
4. Applications
   - Application of the extended ABM
   - Application of the ABM algorithm

Current situation

3D seismic imaging

- Most methods are "model driven"
- Typical result of seismic imaging: model of the subsurface as a cloud of points
- Every point $p \in \mathbb{R}^3$ is associated with a tuple of parameters, typically velocity, density, ...
- Non continuous changes in these parameters allow conclusions about the geological subsurface structure

Current situation
3D seismic imaging

- Most methods are “model driven”
- Typical result of seismic imaging: model of the subsurface as a cloud of points
- Every point $p \in \mathbb{R}^3$ is associated with a tuple of parameters, typically velocity, density, ...
- Non continuous changes in these parameters allow conclusions about the geological subsurface structure

Current situation

3D seismic imaging

- Most methods are "model driven"
- Typical result of seismic imaging: model of the subsurface as a cloud of points
- Every point $p \in \mathbb{R}^3$ is associated with a tuple of parameters, typically velocity, density, ...
- Non continuous changes in these parameters allow conclusions about the geological subsurface structure

Current situation
3D seismic imaging

- Most methods are “model driven”
- Typical result of seismic imaging: model of the subsurface as a cloud of points
- Every point $p \in \mathbb{R}^3$ is associated with a tuple of parameters, typically velocity, density, ...
- Non continuous changes in these parameters allow conclusions about the geological subsurface structure

Current situation
Interpretation of seismic images

- Introduction of layers/regions, which allow the separation of different regions
  - usually performed in an interactive way
  - rather simple geometric structures
  - or composition of local approximations e.g. simplicial surfaces

Current situation

Interpretation of seismic images

- Introduction of layers/regions, which allow the separation of different regions
  - usually performed in an interactive way
  - rather simple geometric structures
  - or composition of local approximations e.g. simplicial surfaces

Current situation

Interpretation of seismic images

- Introduction of layers/regions, which allow the separation of different regions
  - usually performed in an interactive way
  - rather simple geometric structures
  - or composition of local approximations e.g. simplicial surfaces

Current situation

Advantages and disadvantages

Advantages

- Proven techniques
- Incorporates knowledge of geologists/interpreters

Disadvantages

- Only local view of the problem
  - Difficult to relate temporal changes in the oil field (4D seismics)
- Big datasets especially for 4D seismics

Current situation
Advantages and disadvantages

Advantages

- Proven techniques
- Incorporates knowledge of geologists/interpreters

Disadvantages

- Only local view of the problem
  - Difficult to relate temporal changes in the oil field (4D seismics)
- Big datasets especially for 4D seismics

Current situation
Advantages and disadvantages

Advantages

- Proven techniques
- Incorporates knowledge of geologists/interpreters

Disadvantages

- Only local view of the problem
  - Difficult to relate temporal changes in the oil field (4D seismics)
- Big datasets especially for 4D seismics

Current situation
Advantages and disadvantages

Advantages
- Proven techniques
- Incorporates knowledge of geologists/interpreters

Disadvantages
- Only local view of the problem
  - Difficult to relate temporal changes in the oil field (4D seismics)
- Big datasets especially for 4D seismics

Current situation
Advantages and disadvantages

Advantages

- Proven techniques
- Incorporates knowledge of geologists/interpreters

Disadvantages

- Only local view of the problem
  - Difficult to relate temporal changes in the oil field (4D seismics)
- Big datasets especially for 4D seismics

Where we come into play
Our vision

Are there alternatives?

Our vision

- Move from a model driven world to a data driven approach
  - Less upfront assumptions
- Compact mathematical representation
- Discover “unconventional” geometric structures

Where we come into play

Our vision

Are there alternatives?

Our vision

- Move from a model driven world to a data driven approach
  - Less upfront assumptions
- Compact mathematical representation
- Discover “unconventional” geometric structures

Where we come into play
Our vision

Are there alternatives?

Our vision

- Move from a model driven world to a data driven approach
  - Less upfront assumptions
- Compact mathematical representation
- Discover “unconventional” geometric structures

Where we come into play
Our vision

Are there alternatives?

Our vision

- Move from a model driven world to a data driven approach
  - Less upfront assumptions
- Compact mathematical representation
- Discover “unconventional” geometric structures

Algebraic surfaces

A known example

Example (3D Unit Sphere)

Equation:

$$x^2 + y^2 + z^2 = 1$$

Algebraic surfaces

Definition (3 dimensional algebraic surface)

The (real) zeroset of a polynomial equation in 3 indeterminates
$\mathscr{Z}(p),\ p \in \mathbb{R}[x, y, z]$

Algebraic surfaces

Advantages in modelling

- Rather "simple" equations
- Compact description compared to simplicial surfaces (≈triangulated surfaces)
- Mathematical theory provided by Algebraic Geometry

Algebraic surfaces
Advantages in modelling

- Rather "simple" equations
- Compact description compared to simplicial surfaces (≈triangulated surfaces)
- Mathematical theory provided by Algebraic Geometry

Algebraic surfaces

Advantages in modelling

- Rather "simple" equations
- Compact description compared to simplicial surfaces (≈triangulated surfaces)
- Mathematical theory provided by Algebraic Geometry

Algebraic surfaces

A basis for future improvements

Algebraic surfaces provide the foundation for relating the changes of the shape of an oil field in time due to

- "Simple" description
- Deformation of algebraic surfaces

Long term goal:

Relate oil/gas production to the changes in the shape of the oil body.

Algebraic surfaces

A basis for future improvements

Algebraic surfaces provide the foundation for relating the changes of the shape of an oil field in time due to

- "Simple" description
- Deformation of algebraic surfaces

Long term goal:

Relate oil/gas production to the changes in the shape of the oil body.

Algebraic surfaces

A basis for future improvements

Algebraic surfaces provide the foundation for relating the changes of the shape of an oil field in time due to

- "Simple" description
- Deformation of algebraic surfaces

Long term goal:

Relate oil/gas production to the changes in the shape of the oil body.

Algebraic surfaces

A basis for future improvements

Algebraic surfaces provide the foundation for relating the changes of the shape of an oil field in time due to

- "Simple" description
- Deformation of algebraic surfaces

Long term goal:

Relate oil/gas production to the changes in the shape of the oil body.

Milestones
Completed tasks

October 2008: Initial involvement in project

October 2008 - February 2009: Getting familiar with AVI and the ideas developed in the project so far

November 2008 - February 2009: Initial ideas for the ABM-algorithm

March 2009 - August 2009: Internship in Rijswijk

September 2009 - February 2010: Developement and implementation of algorithms/ Application to synthetic data

Milestones

Completed tasks

October 2008: Initial involvement in project

October 2008 - February 2009: Getting familiar with AVI and the ideas developed in the project so far

November 2008 - February 2009: Initial ideas for the ABM-algorithm

March 2009 - August 2009: Internship in Rijswijk

September 2009 - February 2010: Developement and implementation of algorithms/ Application to synthetic data

Milestones
Completed tasks

October 2008: Initial involvement in project
October 2008 - February 2009: Getting familiar with AVI and the ideas developed in the project so far
November 2008 - February 2009: Initial ideas for the ABM-algorithm
March 2009 - August 2009: Internship in Rijswijk
September 2009 - February 2010: Developement and implementation of algorithms/ Application to synthetic data

Milestones
Completed tasks

October 2008: Initial involvement in project
October 2008 - February 2009: Getting familiar with AVI and the ideas developed in the project so far
November 2008 - February 2009: Initial ideas for the ABM-algorithm
March 2009 - August 2009: Internship in Rijswijk
September 2009 - February 2010: Developement and implementation of algorithms/ Application to synthetic data

Milestones
Completed tasks

October 2008: Initial involvement in project
October 2008 - February 2009: Getting familiar with AVI and the ideas developed in the project so far
November 2008 - February 2009: Initial ideas for the ABM-algorithm
March 2009 - August 2009: Internship in Rijswijk
September 2009 - February 2010: Developement and implementation of algorithms/ Application to synthetic data

Milestones

Future tasks

March 2010 - September 2010: Application to real world seismic data

September 2010 - March 2011: Investigation of relation between production and change of shapes

March 2010 - September 2011: Completion and compilation of the thesis

Milestones
Future tasks

March 2010 - September 2010: Application to real world seismic data

September 2010 - March 2011: Investigation of relation between production and change of shapes

March 2010 - September 2011: Completion and compilation of the thesis

Milestones

Future tasks

March 2010 - September 2010: Application to real world seismic data

September 2010 - March 2011: Investigation of relation between production and change of shapes

March 2010 - September 2011: Completion and compilation of the thesis

Starting point - AVI

Getting familiar with AVI

Input

Set of noisy measurements $\mathbb{X} = \{p_1, ..., p_\mu\} \in \mathbb{R}^d$ and threshold number ε.

Output

Approximate border basis G with respect to an order ideal $\mathcal{O}$.

Features of the original AVI algorithm

- Approximate border basis
- Set of polynomials, which have small evaluations at $\mathbb{X}$

Starting point - AVI

Getting familiar with AVI

Input

Set of noisy measurements $\mathbb{X} = \{p_1, \ldots, p_\mu\} \in \mathbb{R}^d$ and threshold number ε.

Output

Approximate border basis G with respect to an order ideal $\mathcal{O}$.

Features of the original AVI algorithm

- Approximate border basis
- Set of polynomials, which have small evaluations at $\mathbb{X}$

Starting point - AVI

Getting familiar with AVI

Input

Set of noisy measurements $\mathbb{X} = \{p_1, ..., p_\mu\} \in \mathbb{R}^d$ and threshold number ε.

Output

Approximate border basis G with respect to an order ideal $\mathcal{O}$.

Features of the original AVI algorithm

- Approximate border basis
- Set of polynomials, which have small evaluations at $\mathbb{X}$

Internship in Rijswijk

Goals

- Learn the very basics of seismics and seismic imaging
- Get in contact with people working on seismics
- Acquire capability to work with the conventional seismic toolchain ⇒ Seismic Unix

Internship in Rijswijk

Goals

- Learn the very basics of seismics and seismic imaging
- Get in contact with people working on seismics
- Acquire capability to work with the conventional seismic toolchain $\Rightarrow$ Seismic Unix

Internship in Rijswijk

Goals

- Learn the very basics of seismics and seismic imaging
- Get in contact with people working on seismics
- Acquire capability to work with the conventional seismic toolchain $\Rightarrow$ Seismic Unix

Outline

1 Motivation
- General Setting

2 Timeline

3 Developement and implementation of algorithms
- ABM algorithm
- The extended ABM algorithm

4 Applications
- Application of the extended ABM
- Application of the ABM algorithm

The ABM algorithm

Algorithm (ABM)

Let $\mathbb{X} = \{p_1, ..., p_s\} \subset \mathbb{R}^n$, let $P = \mathbb{R}[x_1, ..., x_n]$, let $eval : P \to \mathbb{R}^s$ be the associated evaluation map $eval(f) = (f(p_1), ..., f(p_s))$ and let $\varepsilon > \tau > 0$ be small numbers. With $\|\cdot\|$ we denote the euclidean norm. Moreover we choose a degree compatible term ordering σ.

1. *Start with lists $G = \emptyset$, $\mathcal{O} = [1]$, a matrix $M = (1, ..., 1)^{tr} \in Mat_{s,1}(\mathbb{R})$ and $d = 0$.*
2. *Increase d by one and let L be the list of terms in degree d in $\partial\mathcal{O}$ ordered decreasingly with respect to σ. If $L = \emptyset$ return the pair $(G, \mathcal{O})$ and stop. Otherwise let $L = (t_1, ..., t_l)$.*
3. *Begin with $i := 1$ and calculate*
$$A = eval(t_i, M) \in Mat_{s,1+m}(\mathbb{R})$$
4. *Now calculate the least squares solution of $Ax \approx \vec{0}$ with $\|x\| = 1$, which is the smallest norm one eigenvector of $A^{tr}A$. Let us denote the solution with $s = (s_1, ..., s_l)$ and the smallest eigenvector with e.*
5. *Now calculate $\iota = \sqrt{e}$ and check if $\iota < \varepsilon$. If so, we add $s_1 t_1 + ... + s_l t_l = 0$ to G, otherwise we add t_i to the order ideal $\mathcal{O}$ and additionally $eval(t_i)$ to M.*
6. *Now we set $i := i + 1$. As long as $i < l$ go to step 3.*
7. *Continue with step 2.*

The ABM algorithm

Differences between AVI and ABM

- Term by term (ABM) versus degree by degree (AVI)
- Different approach to calculate the polynomials. SVD in AVI, eigenvectors in ABM
- Less need for scaling of input data
- Direct error measure ε (ABM) versus indirect error measure (AVI)

The last property is important to control the "fit" of the algebraic surface

The ABM algorithm

Differences between AVI and ABM

- Term by term (ABM) versus degree by degree (AVI)
- Different approach to calculate the polynomials. SVD in AVI, eigenvectors in ABM
- Less need for scaling of input data
- Direct error measure ε (ABM) versus indirect error measure (AVI)

The last property is important to control the “fit” of the algebraic surface

The ABM algorithm

Differences between AVI and ABM

- Term by term (ABM) versus degree by degree (AVI)
- Different approach to calculate the polynomials. SVD in AVI, eigenvectors in ABM
- Less need for scaling of input data
- Direct error measure ε (ABM) versus indirect error measure (AVI)

The last property is important to control the “fit” of the algebraic surface

The ABM algorithm

Differences between AVI and ABM

- Term by term (ABM) versus degree by degree (AVI)
- Different approach to calculate the polynomials. SVD in AVI, eigenvectors in ABM
- Less need for scaling of input data
- Direct error measure ε (ABM) versus indirect error measure (AVI)

The last property is important to control the "fit" of the algebraic surface

The ABM algorithm

Differences between AVI and ABM

- Term by term (ABM) versus degree by degree (AVI)
- Different approach to calculate the polynomials. SVD in AVI, eigenvectors in ABM
- Less need for scaling of input data
- Direct error measure ε (ABM) versus indirect error measure (AVI)

The last property is important to control the “fit” of the algebraic surface

Outline

1. Motivation
   - General Setting
2. Timeline
3. Developement and implementation of algorithms
   - ABM algorithm
   - The extended ABM algorithm
4. Applications
   - Application of the extended ABM
   - Application of the ABM algorithm

The extended ABM algorithm

Algorithm (Extended ABM)

Let $\mathbb{X} = \{p_1, ..., p_s\} \subset \mathbb{R}^n$, $\mathbb{V} = (v_1, ..., v_s) \subset \mathbb{R}$ let $P = \mathbb{R}[x_1, ..., x_{n+1}]$, let $eval : P \to \mathbb{R}^s$ be the associated evaluation map $eval(f) = (f(p_1), ..., f(p_s))$ and let $\varepsilon > \tau > 0$ be small numbers. With $\|\cdot\|$ we denote the euclidean norm. Moreover we choose a degree compatible term ordering σ.

1. *Start with lists $G = \emptyset$, $\mathcal{O} = [1]$, a matrix $M = (1, ..., 1)^{tr} \in Mat_{s,1}(\mathbb{R})$ and $d = 0$.*
2. *Increase d by one and let L be the list of terms in degree d in $\partial\mathcal{O}$ ordered decreasingly with respect to σ. If $L = \emptyset$ return the pair $(G, \mathcal{O} \cup x_{n+1})$ and stop. Otherwise let $L = (t_1, ..., t_l)$.*
3. *Begin with $i := 1$ and calculate*

$$A = eval(t_i, M) \in Mat_{s,1+m}(\mathbb{R})$$

4. *Now calculate the least squares solution of $Ax \approx \mathbb{V}$. If $\|\mathbb{V}\| < \tau$ calculate the solution of $Ax \approx \vec{0}, \|x\| = 1$, which is the smallest norm one eigenvector of $A^{tr}A$. If $\|\mathbb{V}\| \geq \tau$ calculate it using the QR decomposition of A. Let us denote the solution with $s = (s_1, ..., s_l)$.*
5. *Now calculate $\iota = \|As - \mathbb{V}\|$ and check if $\iota < \varepsilon$. If so, we add $s_1 t_1 + ... + s_l t_l - x_{n+1} = 0$ to G, otherwise we add t_i to the order ideal $\mathcal{O}$ and additionally $eval(t_i)$ to M.*
6. *Now we set $i := i + 1$. As long as $i < l$ go to step 3.*
7. *Continue with step 2.*

The extended ABM algorithm
Properties

Properties

- ABM is a special case of the extended ABM algorithm
- Allows the controlled modelling of one specific measurement
  - Resulting polynomials are functions in the input measurements
  - Direct control of the residual error

The extended ABM algorithm

Properties

Properties

- ABM is a special case of the extended ABM algorithm
- Allows the controlled modelling of one specific measurement
  - Resulting polynomials are functions in the input measurements
  - Direct control of the residual error

Implementation of algorithms

Implementation in ApCoCoALib

- All presented algorithms are implemented in the ApCoCoA library
- Access via
  - the CoCoAL language for prototyping
  - C++ for high performance

Implementation of algorithms

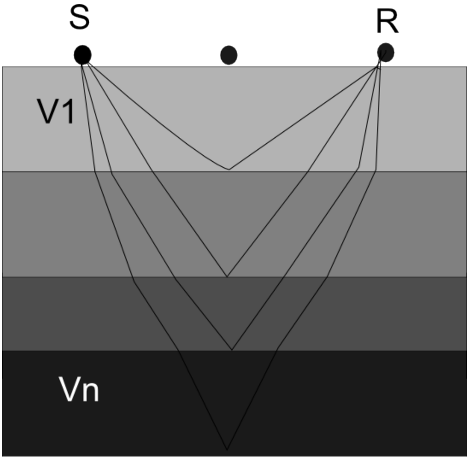
Implementation in ApCoCoALib

- All presented algorithms are implemented in the ApCoCoA library
- Access via
  - the CoCoAL language for prototyping
  - C++ for high performance

Outline

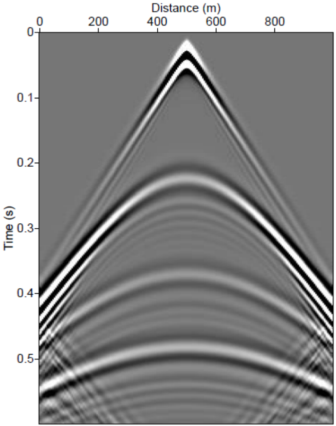
1. Motivation
   - General Setting
2. Timeline
3. Developement and implementation of algorithms
   - ABM algorithm
   - The extended ABM algorithm
4. Applications
   - Application of the extended ABM
   - Application of the ABM algorithm

Recovery of velocities



Recovery of velocities

Similar example: Seismogram for 3 layers



Surface data (dx=5 m, z=5 m)

Recovery of velocities
A data driven approach

Use the extended ABM to model the hyperbolas.

Input: Points picked along a wavefront
Output: Algebraic equations

In the simple case of parallel horizontal layers it is even possible to "read off" the velocities directly from the equations.

Recovery of velocities

A data driven approach

Use the extended ABM to model the hyperbolas.

Input: Points picked along a wavefront
Output: Algebraic equations

In the simple case of parallel horizontal layers it is even possible to "read off" the velocities directly from the equations.

Recovery of velocities

A data driven approach

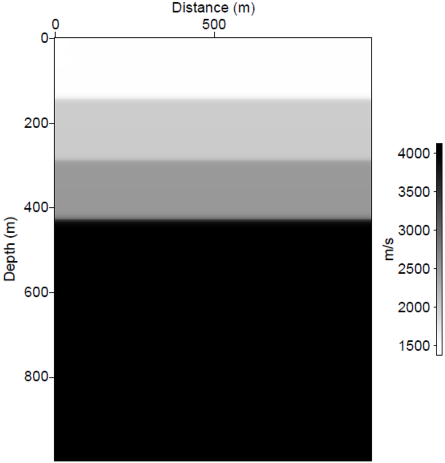
Use the extended ABM to model the hyperbolas.

Input: Points picked along a wavefront
Output: Algebraic equations

In the simple case of parallel horizontal layers it is even possible to "read off" the velocities directly from the equations.

Recovery of velocities

In the case of our example we obtain:



Recovery of velocities

Advantages

- No model assumed upfront
- The model was derived **after** all waves turned out to be hyperbolas
- Insensitive to noise
- Good way to check validity of model

Open problems

- Better interpretation of derived equations in more complex cases
- Application to real world data

Recovery of velocities

Advantages

- No model assumed upfront
- The model was derived **after** all waves turned out to be hyperbolas
- Insensitive to noise
- Good way to check validity of model

Open problems

- Better interpretation of derived equations in more complex cases
- Application to real world data

Recovery of velocities

Advantages

- No model assumed upfront
- The model was derived **after** all waves turned out to be hyperbolas
- Insensitive to noise
- Good way to check validity of model

Open problems

- Better interpretation of derived equations in more complex cases
- Application to real world data

Recovery of velocities

Advantages

- No model assumed upfront
- The model was derived **after** all waves turned out to be hyperbolas
- Insensitive to noise
- Good way to check validity of model

Open problems

- Better interpretation of derived equations in more complex cases
- Application to real world data

Recovery of velocities

Advantages

- No model assumed upfront
- The model was derived **after** all waves turned out to be hyperbolas
- Insensitive to noise
- Good way to check validity of model

Open problems

- Better interpretation of derived equations in more complex cases
- Application to real world data

Recovery of velocities

Advantages

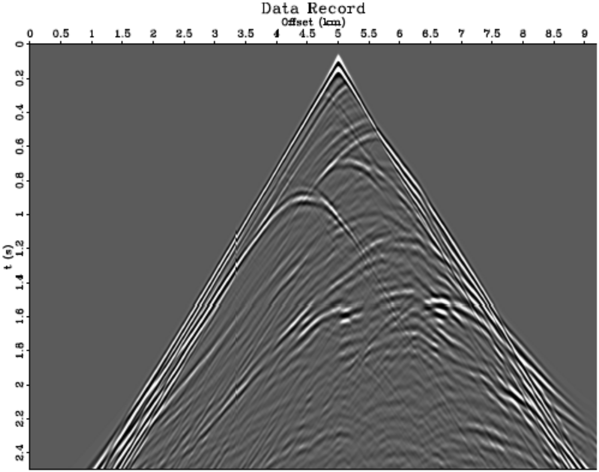
- No model assumed upfront
- The model was derived **after** all waves turned out to be hyperbolas
- Insensitive to noise
- Good way to check validity of model

Open problems

- Better interpretation of derived equations in more complex cases
- Application to real world data

Recovery of velocities

One of our goals - the Marmousi model



Outline

1. Motivation
   - General Setting
2. Timeline
3. Developement and implementation of algorithms
   - ABM algorithm
   - The extended ABM algorithm
4. Applications
   - Application of the extended ABM
   - Application of the ABM algorithm

The recovery of a complex geometric structure
Using noisy incomplete datasets

Example (Recovery of a torus)

- Represents a non conventional geometry/ hard to model with traditional methods
- We use a set of 400 3D points, symbolizing the result of the seismic imaging process
- Points contain up to 20% noise
- Are not dense at all locations

The recovery of a complex geometric structure

Using noisy incomplete datasets

Example (Recovery of a torus)

- Represents a non conventional geometry/ hard to model with traditional methods
- We use a set of 400 3D points, symbolizing the result of the seismic imaging process
- Points contain up to 20% noise
- Are not dense at all locations

The recovery of a complex geometric structure

Using noisy incomplete datasets

Example (Recovery of a torus)

- Represents a non conventional geometry/ hard to model with traditional methods
- We use a set of 400 3D points, symbolizing the result of the seismic imaging process
- Points contain up to 20% noise
- Are not dense at all locations

The recovery of a complex geometric structure

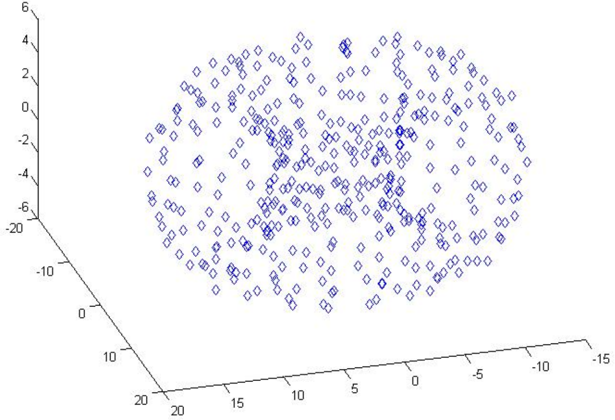
Using noisy incomplete datasets

Example (Recovery of a torus)

- Represents a non conventional geometry/ hard to model with traditional methods
- We use a set of 400 3D points, symbolizing the result of the seismic imaging process
- Points contain up to 20% noise
- Are not dense at all locations

The recovery of a complex geometric structure

Visualized with MatLab



The recovery of a complex geometric structure

1. Convert input data into CoCoA matrix format (list format)
2. Apply ABM algorithm with $\varepsilon = 4$. Runtime 1.4 seconds for 400 Points.
3. The algorithm returns a set of 45 polynomials with respect to an order ideal of size 75.
   Polynomials are ordered by increasing degree.
   Simpler structures come first.
4. Already the first polynomial gives a reasonable answer!

The recovery of a complex geometric structure

1. Convert input data into CoCoA matrix format (list format)
2. Apply ABM algorithm with $\varepsilon = 4$. Runtime 1.4 seconds for 400 Points.
3. The algorithm returns a set of 45 polynomials with respect to an order ideal of size 75.
   Polynomials are ordered by increasing degree.
   Simpler structures come first.
4. Already the first polynomial gives a reasonable answer!

The recovery of a complex geometric structure

1. Convert input data into CoCoA matrix format (list format)
2. Apply ABM algorithm with $\varepsilon = 4$. Runtime 1.4 seconds for 400 Points.
3. The algorithm returns a set of 45 polynomials with respect to an order ideal of size 75.
   Polynomials are ordered by increasing degree.
   Simpler structures come first.
4. Already the first polynomial gives a reasonable answer!

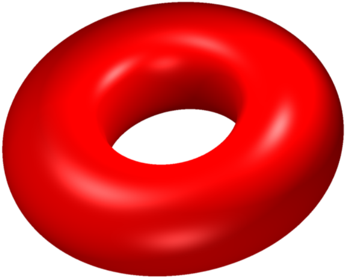
The recovery of a complex geometric structure

1. Convert input data into CoCoA matrix format (list format)
2. Apply ABM algorithm with $\varepsilon = 4$. Runtime 1.4 seconds for 400 Points.
3. The algorithm returns a set of 45 polynomials with respect to an order ideal of size 75.
   Polynomials are ordered by increasing degree.
   Simpler structures come first.
4. Already the first polynomial gives a reasonable answer!

The recovery of a complex geometric structure

Visualization of the first polynomial

$$x^4 + 2x^2y^2 + y^4 + 2.3x^2z^2 + 2.2y^2z^2 - 276.8x^2 - 278.2y^2 + 10822.8$$



The recovery of a complex geometric structure

So how good is the obtained result?

Original equation:

$$x^4 + 2x^2y^2 + y^4 + 2x^2z^2 + 2y^2z^2 + z^4 - 250x^2 - 250y^2 + 150z^2 + 5625$$

Compared to recovered equation:

$$x^4 + 2x^2y^2 + y^4 + 2.3x^2z^2 + 2.2y^2z^2 - 276.8x^2 - 278.2y^2 + 10822.8$$

Given only a set of noisy measurements we were able to recover a slightly deformed torus!

The recovery of a complex geometric structure

So how good is the obtained result?

Original equation:

$$x^4+2x^2y^2+y^4+2x^2z^2+2y^2z^2+z^4-250x^2-250y^2+150z^2+5625$$

Compared to recovered equation:

$$x^4+2x^2y^2+y^4+2.3x^2z^2+2.2y^2z^2-276.8x^2-278.2y^2+10822.8$$

Given only a set of noisy measurements we were able to recover a slightly deformed torus!

The recovery of a complex geometric structure

So how good is the obtained result?

Original equation:

$$x^4+2x^2y^2+y^4+2x^2z^2+2y^2z^2+z^4-250x^2-250y^2+150z^2+5625$$

Compared to recovered equation:

$$x^4+2x^2y^2+y^4+2.3x^2z^2+2.2y^2z^2-276.8x^2-278.2y^2+10822.8$$

Given only a set of noisy measurements we were able to recover a slightly deformed torus!

The recovery of a complex geometric structure

Advantages

- No shape assumed upfront
- Recovery of a non conventional structure
- Compact representation: 400 points reduced to one equation

The recovery of a complex geometric structure

Advantages

- No shape assumed upfront
- Recovery of a non conventional structure
- Compact representation: 400 points reduced to one equation

The recovery of a complex geometric structure

Advantages

- No shape assumed upfront
- Recovery of a non conventional structure
- Compact representation: 400 points reduced to one equation

Summary

- Overview of current situation
- (Extended) ABM algorithm
- Application to example data

- Outlook
  - Apply algorithm to more complex/real data
  - Improve algorithm(s) and surrounding toolchain
  - Prove mathematical correctness of algorithms in PhD thesis

Summary

- Overview of current situation
- (Extended) ABM algorithm
- Application to example data
- Outlook
  - Apply algorithm to more complex/real data
  - Improve algorithm(s) and surrounding toolchain
  - Prove mathematical correctness of algorithms in PhD thesis

Summary

- Overview of current situation
- (Extended) ABM algorithm
- Application to example data

- Outlook
    - Apply algorithm to more complex/real data
    - Improve algorithm(s) and surrounding toolchain
    - Prove mathematical correctness of algorithms in PhD thesis

Summary

- Overview of current situation
- (Extended) ABM algorithm
- Application to example data

- Outlook
  - Apply algorithm to more complex/real data
  - Improve algorithm(s) and surrounding toolchain
  - Prove mathematical correctness of algorithms in PhD thesis

Summary

- Overview of current situation
- (Extended) ABM algorithm
- Application to example data

- Outlook
  - Apply algorithm to more complex/real data
  - Improve algorithm(s) and surrounding toolchain
  - Prove mathematical correctness of algorithms in PhD thesis

Summary

- Overview of current situation
- (Extended) ABM algorithm
- Application to example data

- Outlook
    - Apply algorithm to more complex/real data
    - Improve algorithm(s) and surrounding toolchain
    - Prove mathematical correctness of algorithms in PhD thesis

Thank you for your attention

Any questions?